Chapter 5
Fourier Analysis

5.1 Introduction

The theory, practice, and application of Fourier analysis are presented in the three major sections of this chapter. The theory includes a discussion of Fourier transforms, including the Fourier integral and the discrete Fourier transform, along with suggestions on computing the spectrum of quasiperiodic waveforms. The section on practice presents details on error mechanisms as well as on how Fourier analysis is implemented in SPICE, Spectre, and external Fourier analyzers. Suggestions are given on how to achieve accurate results. The chapter concludes by presenting various types of circuits that present challenges to Fourier analysis and how those challenges might be addressed.

5.1.1 Distortion

Fourier analysis is almost exclusively used to measure distortion. When measuring distortion, there are two important cases to consider: wide-band and narrow-band circuits.

5.1.1.1 Harmonic Distortion

To measure the distortion of wide-band circuits, one simply applies a pure single-tone sine wave to the input, and determines how much

the output deviates from being a pure sinusoid after the circuit has reached steady state. The distortion products in the output signal fall at frequencies that are integer multiples (harmonics) of the input frequency. In this case, the distortion measure of choice is *total harmonic distortion* or *THD*.

Total Harmonic Distortion
THD is a measure of the amount of energy in the harmonics relative to the energy in the fundamental.

$$\text{THD} = \sqrt{\frac{\sum_{k=2}^{\infty} |X_k|^2}{|X_1|^2}} \cdot 100\% \qquad (5.1)$$

where $|X_k|$ is the magnitude of the kth harmonic.

5.1.1.2   Intermodulation Distortion

If this same technique were applied to measure the distortion of a narrow-band amplifier, the harmonic distortion products would normally fall outside the bandwidth of the amplifier and so would be attenuated. The amount of distortion calculated using (5.1) would be much too low.

Distortion is measured in narrow-band amplifiers by applying two pure sinusoids with frequencies well within the bandwidth of the amplifier (call these frequencies f_1 and f_2). The harmonics of these two frequencies are outside the bandwidth of the amplifier, however there are distortion products that fall at the frequencies $2f_1 - f_2$, $2f_2 - f_1$, $3f_1 - 2f_2$, $3f_2 - 2f_1$, etc. As shown in Figure 5.1 on the next page these frequencies should also be well within the bandwidth of the amplifier and so can be used to measure accurately what is referred to as the *intermodulation distortion*, or *IMD*, produced by the amplifier.

5.1.1.3   Compression and Intercept Points

Harmonic and intermodulation distortion both characterize signals rather than the underlying circuit. At low frequencies, it is common
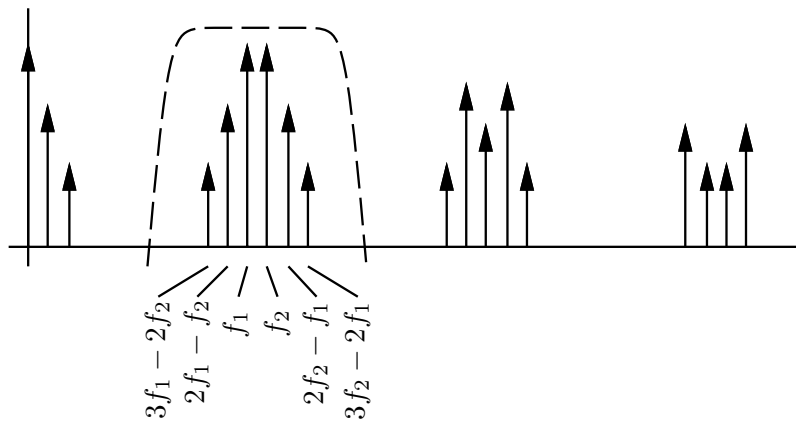
Figure 5.1: Two-tone intermodulation products superimposed on a narrow-band filter's passband.

to describe the distortion of a circuit by indicating the distortion in the output signal when driven by a sinusoid to achieve a certain output level. For example, an audio power amplifier might be described as having 0.1% THD at 100 Watts. At high frequencies it is more common to characterize the distortion produced by a circuit in terms of a compression point or an intercept point. These metrics characterize the circuit rather than the signal, and as such it is not necessary to specify the signal level at which the circuit was characterized.

In order to more easily understand the definition of a compression point and an intercept point, consider the output power of the first three harmonics for a GaAs traveling-wave amplifier, as shown in Figure 5.2 on the following page. The 1 dB compression point is the value of the output power when the gain of the amplifier has dropped 1 dB from it small-signal asymptotic value, as shown in Figure 5.3 on page 255.

The compression point is a difficult measurement to make with a time-domain circuit simulator (SPICE) because one must measure the
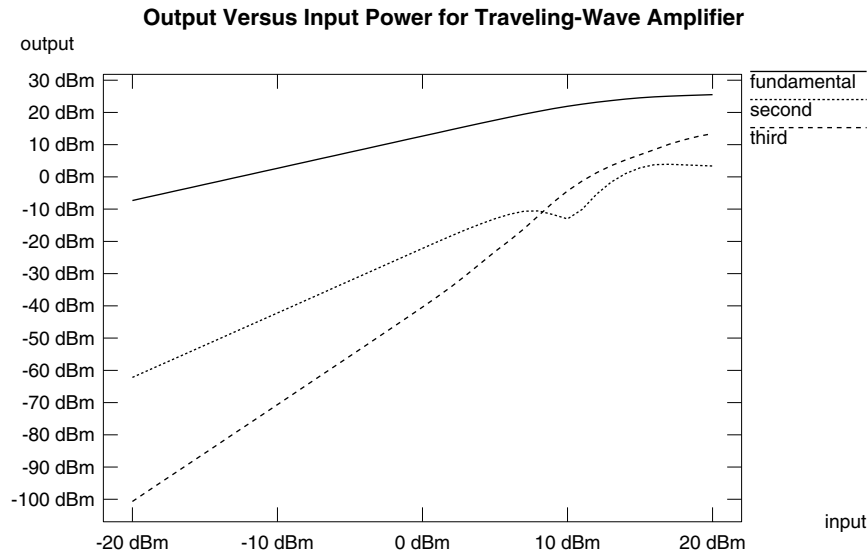
Output Versus Input Power for Traveling-Wave Amplifier

Figure 5.2: The power in the first three harmonics at the output of the traveling-wave amplifier versus the input power.

power in the fundamental of the output (using transient and Fourier analysis) while sweeping the input power. This generally requires many simulation runs. The actual compression point is found by drawing a figure similar to Figure 5.3 and determining the proper value graphically.

The intercept points are defined in terms of the power levels of the various harmonics as extrapolated from their asymptotic small-signal behavior. When the input signal is small, a doubling of the input power doubles the output power at the fundamental, multiplies the output power of the second harmonic by 4 (2^2), and multiplies the output power of the third harmonic by 8 (2^3). Thus, the asymptotic slope of the fundamental is 1 dB/dB, the asymptotic slope of the second harmonic is 2 dB/dB, and the asymptotic slope of the third harmonic is 3 dB/dB. The second-order intercept point (IP_2)
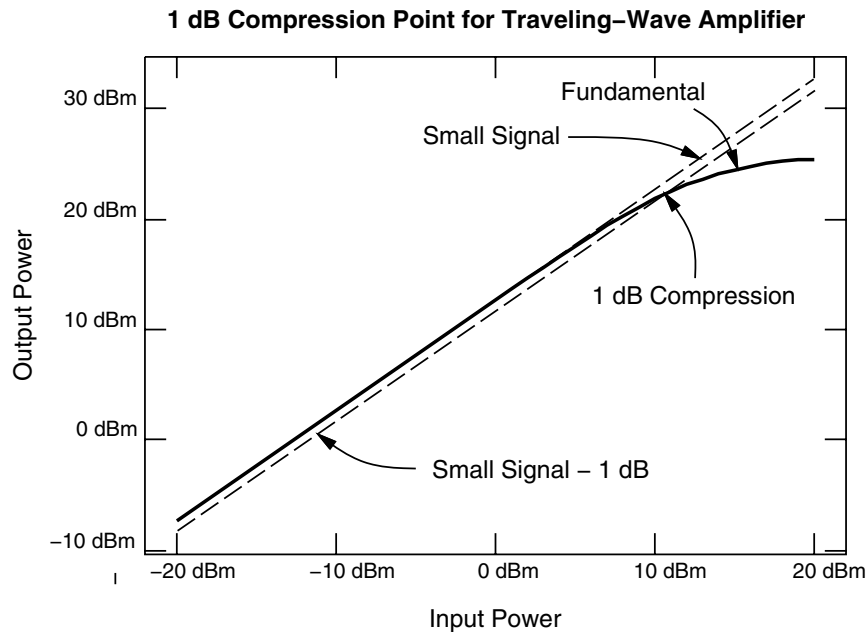
1 dB Compression Point for Traveling–Wave Amplifier

Figure 5.3: The 1 dB compression point is the value of the fundamental output power where it crosses the line that represents the output power extrapolated from small signal conditions minus 1 dB. In this example, the compression point is 22.3 dBm.

is where the asymptotes for the second harmonic and the fundamental cross. Similarly, the third-order intercept point (IP_3) is where the asymptotes for the third harmonic and the fundamental cross. The definitions of IP_2 and IP_3 are illustrated in Figure 5.4 on the following page.

The intercept point is easier to measure with a time-domain circuit simulator than is the compression point because it requires just one transient and one Fourier analysis. Simply choose an input power level that you are confident puts the circuit in the 'small-signal' regime and then measure level of the first, second, and third har-
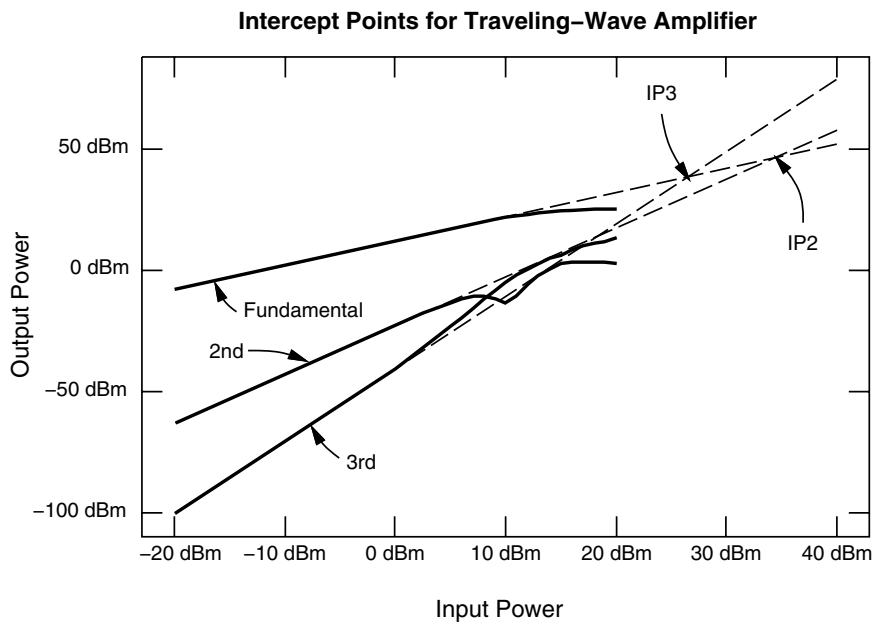
Intercept Points for Traveling–Wave Amplifier

Figure 5.4: The second-order intercept point (IP_2) is the output power where the second-order term as extrapolated from small-signal conditions crosses the extrapolated power of the fundamental. Similarly, the third-order intercept point (IP_3) is the output power where the third-order term as extrapolated from small-signal conditions crosses the extrapolated power of the fundamental. In this example, IP_2 is 47.6 dBm and IP_3 is 39.4 dBm.

monics. Let P_i be the power of the input test signal and P_1, P_2, and P_3 be the power in the first, second, and third harmonics at the output. Assume that the power is measured in your favorite flavor of decibels (dBm, dBW, dBV, etc). Then,

$$IP_2 = 2P_1 - P_2 \qquad (5.2)$$

$$IP_3 = (3P_1 - P_3)/2 \qquad (5.3)$$

These calculations are only accurate if P_i is small enough so that the output power levels are in the asymptotic region for all harmonics, and if P_i is large enough so that the Fourier analysis accurately computes P_1, P_2, and P_3.

In the intercept point definitions just given, harmonic distortion was used for illustrative purposes only. The same definitions are just as easily be applied to intermodulation distortion. In this case, the second-order distortion terms play the role of the second harmonic, and the third-order distortion terms play the role of the third harmonic.

5.2 Fourier Analysis Theory

It is a fundamental tenet in the theory of communications systems that waveforms can be decomposed into a possibly infinite sum of sinusoids or complex exponentials. In a circuit simulator, Fourier analysis is what is used to perform the decomposition. Fourier analysis takes a waveform, which is either a continuous or discrete real function of time, and transforms it into a spectrum, which is either a continuous or discrete complex function of frequency. The value of the spectrum at frequency f is the coefficient of the complex exponential of the same frequency.

5.2.1 The Fourier Series

5.2.1.1 Periodic Waveforms

Periodic waveforms are generated by circuits in steady-state undergoing autonomous oscillation or responding to periodic stimuli (assuming that all stimulus have the same period T). The frequency of

the response is usually equal to the frequency of the stimulus, though occasionally the frequencies of the two are multiples of some common frequency.

If a waveform is T-periodic and continuous, then the waveform can be written as a Fourier series.

$$x(t) = \sum_{k=0}^{\infty} \left[a_k \cos \left(\frac{2\pi kt}{T} \right) + b_k \sin \left(\frac{2\pi kt}{T} \right) \right] \qquad (5.4)$$

A waveform $x(t)$ is T-periodic if $x(t) = x(t - T)$ for all t. The sequence of Fourier coefficients (a_k and b_k) is called the spectrum of x. Periodic waveforms consist of a sum sinusoids at integer multiples of the fundamental frequency $f = 1/T$. The signals at frequencies above the fundamental are called harmonics. Thus, the kth harmonic is the signal at kf.

Given a T-periodic waveform $x(t)$, you compute the Fourier coefficients by evaluating the following Fourier integrals.

$$a_0 = \frac{1}{T} \int_t^{t+T} x(\tau) d\tau \qquad (5.5)$$

$$a_k = \frac{2}{T} \int_t^{t+T} x(\tau) \cos \left(\frac{2\pi k\tau}{T} \right) d\tau \qquad (5.6)$$

$$b_k = \frac{2}{T} \int_t^{t+T} x(\tau) \sin \left(\frac{2\pi k\tau}{T} \right) d\tau \qquad (5.7)$$

The Fourier Integral as a Bank of Receivers   In order to understand how the Fourier integral works, it is useful to view it as a bank of receivers, one for each coefficient that is to be computed, as shown in Figure 5.5 on the next page. Each receiver translates the frequencies of the input signal so that the frequency of interest is moved to DC, and then filters the results in order to eliminate the signals at other frequencies. A comb filter is used that has zeros at the frequency of the fundamental and every harmonic. The comb filter completely eliminates all frequencies that exist in the signal assuming that it is precisely T-periodic except the one at DC. For