An Oscillator Puzzle,

An Experiment in Community Authoring

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Version 2, 1 July 2004

Certain oscillators have been known to exhibit seemingly anomalous phase noise behavior in simulation, but the behavior cannot be fully explained by flaws in the simulation. Rather, it appears to have a rather surprising basis in the circuit itself. This paper attempts to provide a physical basis for this unexpected behavior.

This paper poses the problem and offers a partial explanation. However, rather than wait for every detail to be completely explained before being published, this paper points out places where the explanations are incomplete and asks for help from the readers. This is an experiment in community authoring. The hope is that together we can solve the puzzle and completely document the solution. If you have answers to the questions posed in this paper, please mail them to me, the editor, at *ken@designers-guide.com*.

The first version of this paper was completed in July 2001. It was last updated on May 12, 2006. You can find the most recent version at www.designers-guide.org. Contact the author via e-mail at ken@designers-guide.com.

1 The Puzzle

Consider the differential band-switching oscillator shown in Figure 1. With the band switch transistors turned off (V(C) = 0), Spectre's PNoise simulation of this oscillator predicts the output noise shown in Figure 2, where the output is V(P,N). The noise is higher than expected at low frequencies and it has unusual lumps.

FIGURE 1 *Differential oscillator with band switching.*

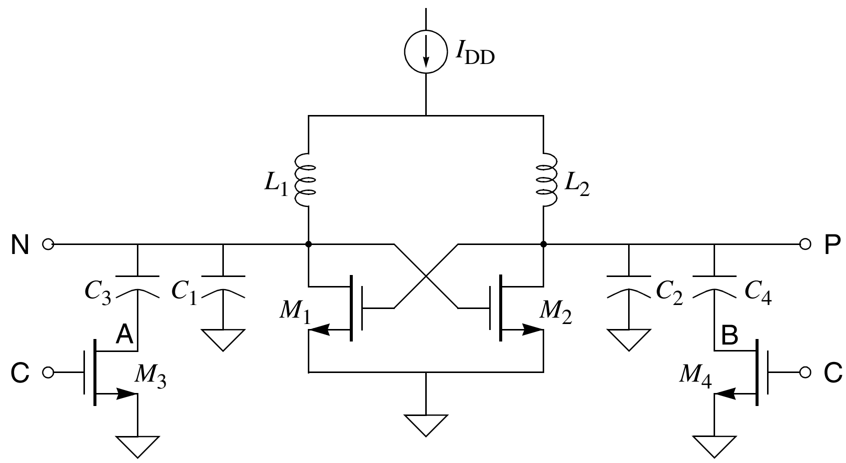
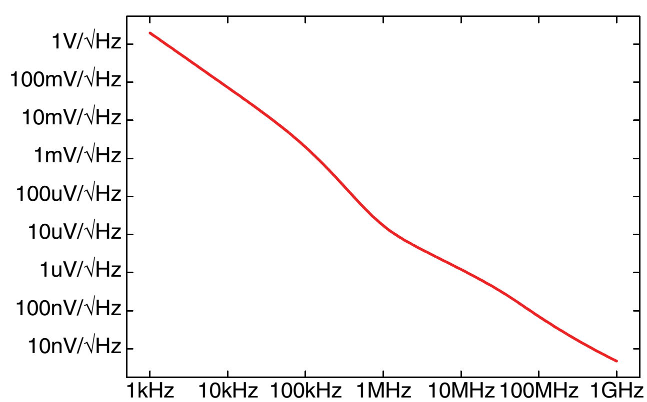


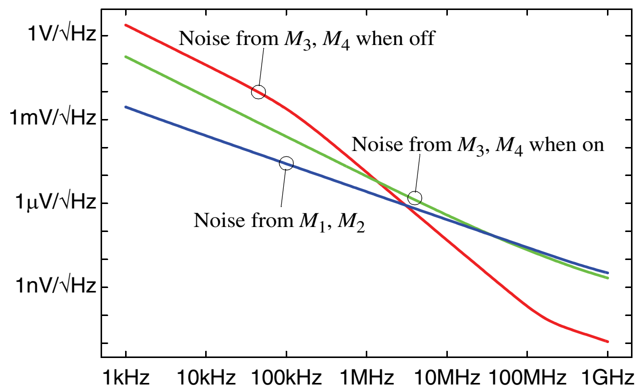
FIGURE 2 *Differential output noise of oscillator.*



2 The Clues

The contributions to the output noise made by the transistors in the oscillator core (M_1 and M_2) and the switch (M_3 and M_4) are shown in Figure 3. The fact that at low frequencies the noise contributed by the switches is so much greater than the noise contributed by the core is the cause of considerable consternation. The perception is that the switches are off, so they should not contribute any noise. In reality, the switches are not completely off. The voltage waveforms at A and B will float down until the transistors turn on briefly during each cycle in reverse subthreshold conduction mode. It is the noise produced by the switch during the brief interval when it is turned on backward that is the source of the noise that causes the problem.

FIGURE 3 *Noise contributions from the core transistors M_1 and M_2 and the switch transistors M_3 and M_4. The noise is given both when the switch transistors are on and off.*



We at Cadence have examined the noise produced within M_3 and M_4 and the levels produced align with expectations. In addition, the noise contribution from M_3 and M_4 to the output grows at a 2½ pole rate from 100 MHz to 100 kHz. These two things cause us to direct our suspicions away from the device noise models and lead us to believe that the transfer function from these noise sources to the output is the cause of the problem.

The transfer function from grounded current sources at nodes B and P to output V(P,N) were computed using a PXF analysis and are shown in Figure 4. These transfer functions assume that the current source is operating at baseband and that the output is near the carrier frequency. Notice that the transfer function from B to the output is much higher than that from P to the output. This is not at all what I would expect if I were to assume that the loading of node B by M_4 was negligible. In this case, the capacitor would be in series with a current source and so could be ignored, and the transfer function from B to the output would be the same as from P to the output. Clearly that is not the case, so the role of M_4 is very important. But it is not simply loading B. If that were the case, it would divert current from C_4 and so the transfer function from B to the output would be less than the transfer function from P to the output. The opposite is true at low frequencies; the transfer function from B is 2500 times larger than that from P.

The transistor model used in our circuit is a subcircuit, shown in Figure 5. I have noticed that if I eliminate all the parasitic components from this subcircuit, the noise amplification problem goes away. If I remove just the substrate resistor, it comes back. If I eliminate all but the gate resistor, some of the amplification comes back, but the noise contributed to the output by the switch transistors remains at least 2 orders of magnitude smaller than the core transistors. Finally, if I remove both the gate and bulk resistors, but leave the diodes, we again have some of the amplification, as shown in Figure 6, but the noise from the switches now dominates that from the core by 1½ orders of magnitude at 1 kHz, as shown in Figure 7.

It is important to note that the junction capacitors normally built into the MOS models have been effectively suppressed by setting their area to be roughly 10^{15} times smaller

FIGURE 4 *Transfer functions from grounded current sources connected at nodes* B *and* P *to the output,* V(P,N). *The transfer functions are from baseband to the carrier.*

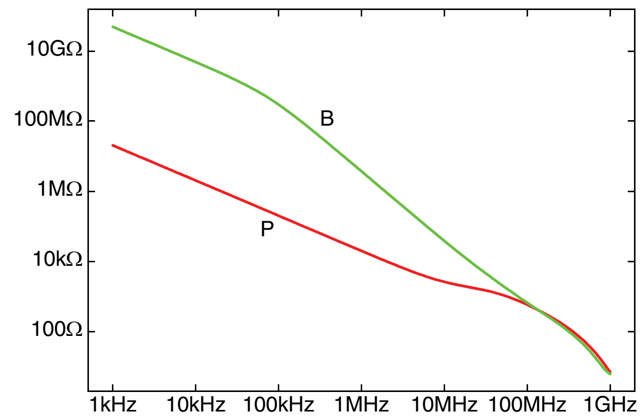


FIGURE 5 *Subcircuit used to model switch and core transistors.*

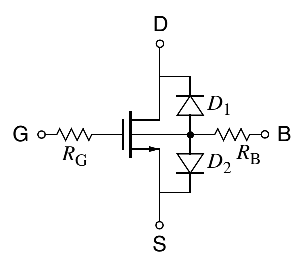
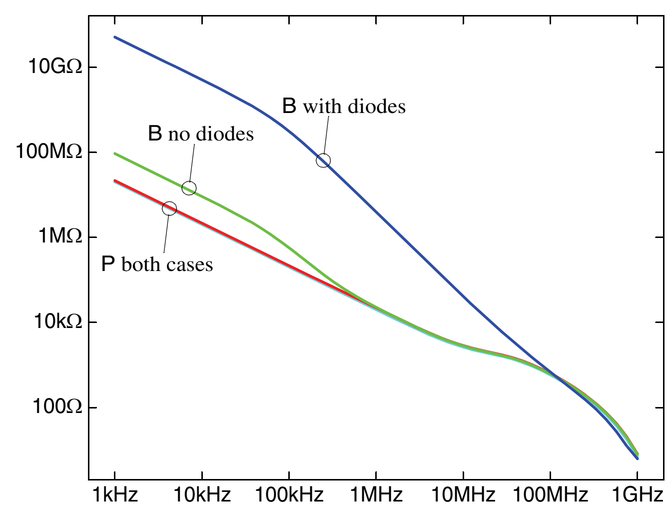
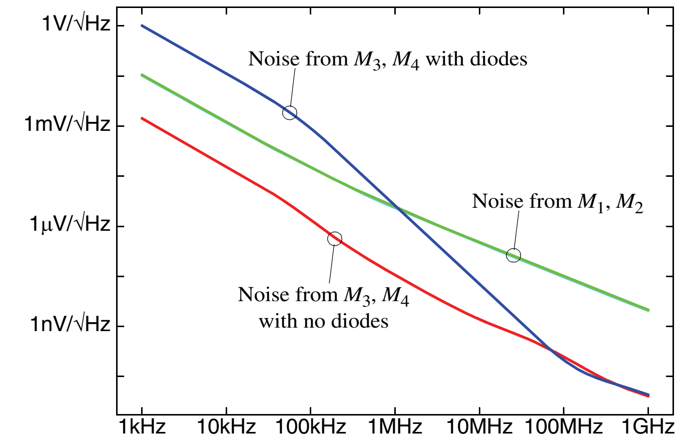


FIGURE 6 *Transfer function when parasitic diodes D_1 and D_2 are removed from all devices.*



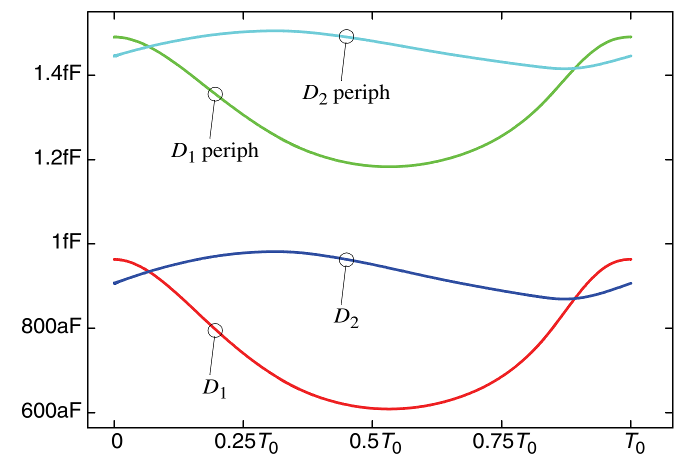
than their actual area. Diving further, we find that when the diodes are present, setting I_S = 0 does not affect the result, but setting C_j = 0 causes the noise amplification to disappear. Similarly, deleting D_2 does not materially affect the result while deleting D_1 causes the noise amplification to disappear again. Thus, the problem seems to stem from

FIGURE 7 *Noise when parasitic diodes D_1 and D_2 are removed from all devices.*



the junction capacitance of D_1. The value of $C_j(t)$ for both the main and periphery junctions for both D_1 and D_2 are shown over time in Figure 8.

FIGURE 8 *Capacitance versus time of the main and periphery junctions for D_1 and D_2 in M_3 and M_4.*



3 A Partial Explanation: Parametric Amplification

I believe that the junction capacitor associated with D_1 is forming a parametric amplifier, which is the mechanism that is amplifying the noise from the switch. A parametric amplifier is an amplifier utilizing a nonlinear reactance, or a reactance that is varied as a function of time by applying a suitable pump signal. The time variation of a reactive parameter, in our case a capacitance, is used to produce the amplification. This is the origin of the term parametric amplification. Parametric amplifiers always involve frequency translation, though the input and output signals need not be at different frequencies. In our case, the input frequency is at baseband and the output frequency is at the upper sideband of the oscillator's fundamental frequency. Parametric amplifiers are desirable because they are capable of very low-noise frequency conversion, as they employ a noise-free mechanism to perform the translation. Ironically, the signal it amplifies in this circuit is the noise signal produced by the MOSFET switch.

With respect to the parametric amplifier, there are two important signals present in this circuit, the oscillation signal, which is periodic, and the noise signal, which is not. Since the noise signal is small, we can consider the noise at each frequency separately. Thus, for the time being, the noise signal will be represented as a small sinusoid parameterized by its frequency. When analyzing parametric amplifiers it is traditional to refer to the large periodic signal as the pump signal, and the signal being amplified, in this case the noise, as simply the signal. Denote the pump frequency as f_p and the signal frequency as f_s. In steady-state, the circuit will respond with signals of the form

$$v(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} V_{mn} e^{j2\pi(mf_\mathrm{p} + nf_\mathrm{s})t} \tag{1}$$

$$i(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_{mn} e^{j2\pi(mf_\mathrm{p} + nf_\mathrm{s})t} \tag{2}$$

where V_{mn} and I_{mn} are the Fourier coefficients of the v and i, and m and n are the harmonic indices of f_p and f_s.

Consider a lossless voltage-controlled nonlinear capacitor. A lossless capacitor can store energy, but can neither generate nor dissipate energy. As such, in steady-state, the total power entering such a capacitor over all frequencies must sum to zero.

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} P_{mn} = 0 \tag{3}$$

where $P_{mn} = V_{mn} I_{nm}^* + V_{mn}^* I_{nm}$. Parametric amplifiers are governed by the Manley-Rowe equations, which are an extension of (3). They are derived by multiplying both sides of (3) by $(mf_\mathrm{p} + nf_\mathrm{s})/(mf_\mathrm{p} + nf_\mathrm{s})$ and then splitting the sum into two parts

$$f_\mathrm{p} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{mn}}{mf_\mathrm{p} + nf_\mathrm{s}} + f_\mathrm{s} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{nP_{mn}}{mf_\mathrm{p} + nf_\mathrm{s}} = 0 \tag{4}$$

Manley and Rowe showed that each double sum vanishes separately [1]. To see why this should be true, rewrite (4) as

$$f_\mathrm{p} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m(V_{mn} I_{mn} + V_{mn} I_{mn}^*)}{mf_\mathrm{p} + nf_\mathrm{s}} + f_\mathrm{s} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{n(V_{mn} I_{mn} + V_{mn} I_{mn}^*)}{mf_\mathrm{p} + nf_\mathrm{s}} = 0 \tag{5}$$

and replace $I_{mn}/(mf_\mathrm{p} + nf_\mathrm{s})$ by $j2\pi Q_{mn}$ to give

$$j2\pi f_{\mathrm{p}} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(V_{mn} Q_{mn} + V_{mn} Q_{mn}{}^*) \; + \tag{6}$$
$$j2\pi f_{\mathrm{s}} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} n(V_{mn} Q_{mn} + V_{mn} Q_{mn}{}^*) \; = \; 0$$

Notice that the arguments to the summation operators do not depend explicitly on either f_{p} or f_{s}. Furthermore, recall that this equation was derived only using the fact that the capacitor is lossless and so must be true for any circuit in which the capacitor is embedded. Consider a particular circuit and f_{p} and f_{s} and find V_{mn} and Q_{mn}. The lossless nature of the capacitor causes V_{mn} and Q_{mn} to satisfy (6) for all m and n. Then choose a new f_{p} and f_{s} and adjust the circuit so that V_{mn} is the same as it was in the first circuit. Since the capacitor is voltage controlled, if V_{mn} is the same, then so too must be Q_{mn}. In this case, f_{p} and f_{s} have changed, but the sums did not. Thus, the only way for the two terms in (6) to sum to zero is if each of the sums are individually zero, and so

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{mn}}{mf_{\mathrm{p}} + nf_{\mathrm{s}}} \; = \; 0 \tag{7}$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{nP_{mn}}{mf_{\mathrm{p}} + nf_{\mathrm{s}}} \; = \; 0 \tag{8}$$

These are the Manley-Rowe relations.

Question 1: This argument provides little physical insite. Is there a simple physical explanation as to why the Manley-Row relations must be true?

Consider a circuit that is driven by sources that only generate power at f_{p} and f_{s}, and for which the load is filtered so that it only absorbs power at $f_{\mathrm{o}} = f_{\mathrm{p}} + f_{\mathrm{s}}$. Then the only power terms that are nonzero are P_{01}, P_{10}, and P_{11}. Thus, for this example, (7) and (8) become

$$\frac{P_{10}}{f_{\mathrm{p}}} + \frac{P_{11}}{f_{\mathrm{p}} + f_{\mathrm{s}}} \; = \; 0 \tag{9}$$

$$\frac{P_{01}}{f_{\mathrm{s}}} + \frac{P_{11}}{f_{\mathrm{p}} + f_{\mathrm{s}}} \; = \; 0 \tag{10}$$

The power gain can now be computed from the input (P_{01}) to the output (P_{11}) using (10) as

$$\frac{P_{11}}{P_{01}} \; = \; -\frac{f_{\mathrm{p}} + f_{\mathrm{s}}}{f_{\mathrm{s}}} \; = \; -\frac{f_{\mathrm{o}}}{f_{\mathrm{s}}} \tag{11}$$

Thus, the power gain equals the ratio of the output to the input frequency. This is an idealized circuit, and so in practice (11) represents an upper bound on the power gain.

Power being absorbed at harmonics and sidebands act to reduce the gain, as will losses in the capacitor.

Question 2: Is there a simple physical explanation for this behavior?

The circuit of Figure 1 fits the assumptions made above. The signal is applied with a current source, which will only generate or consume power at its operating frequency. Both the pump and the output signals are filtered by the resonator, which tends to reject power at other frequencies. Thus this circuit has the potential to be a fairly efficient parametric amplifier. With (11) the maximum gain is found to be 12 MW/W, or 70 dB. This is very close to the roughly 65 dB of gain necessary to explain the results shown in Figure 3 and Figure 4. Furthermore, (11) explains the 2½ pole slope of the phase noise at low offset frequencies. Of the 2½ poles, one pole is inherent to oscillator phase noise, ½ pole is due to the flicker noise source, and the last, previously unexplained pole, is due to the gain of the parametric amplfier having a 1 pole slope.

In summary, the junction capacitor associated with D_1 for the switch is acting as a parametric amplifier and providing almost 65 dB of gain. This amount of gain neatly explains the high levels of noise contributed by the switch to the output.

4 Recommendations

The high levels of noise contributed by the switch in the Spectre simulations appear to be real. However, this does not imply that you will necessarily see this level of noise if you build and measure the oscillator. The parametric amplification is occurring in parasitic components that might not be particularly well modeled. For example, series resistance in the junction capacitance will reduce the gain.

For reasons that are not well understood, the flicker noise model seems dramatically affect the results. The noise produced by the default I_{ds}-based flicker noise model (*flkmod*=0) is strongly amplified, whereas the noise produced by the g_m-based model (*flkmod*=1) is not. It is not clear why the result from the two models should be so different, other than the time-distribution of the noise being somewhat different. However, the g_m-based model is widely regarded as the more accurate model. We recommend use of this model.

Question 3: Why would the g_m-based model exhibit less noise in this case?

If the circuit does exhibit high levels of noise from the switches, it may be possible to change the circuit fix the problem. We found that the amplification was greatly reduced by bypassing the switch with at 10 MΩ resistor. Adding a resistor in series with the gate of the switch might have a similar affect.

I've noticed that the circuit uses a nonlinear $C \times V$ models for the varactor. This will result in unexpected effective capacitance values. Instead, the varactor model should be formulated in terms of charge so that it conserves charge and exhibits the expected capacitance. Do this by integrating the capacitance as a function of v to compute the charge, and then computing the current by differentiating the charge with respect to time [2].

5 Results

Using models which specified the g_m-based flicker noise model (*flkmod*=1) rather than the I_{ds}-based model (*flkmod*=0), and for which the *kf*, *af*, and *ef* parameters were updated appropriately, resulted in the noise contributions shown in Figure 9, While the noise contribution from M_3 and M_4 still exhibits parametric amplification when the switches are off, the contribution is considerably smaller than the noise contributed by M_1 and M_2. Comparing against Figure 3 shows that with the new noise models, M_1 and M_2 are contributing somewhat more noise and M_3 and M_4 are contributing considerably less. The total output noise is shown in Figure 10,

FIGURE 9 *Noise contributions from the core transistors M_1 and M_2 and the switch transistors M_3 and M_4 when using g_m-based flicker noise models (flkmod=1). The noise is given both when the switch transistors are on and off.*

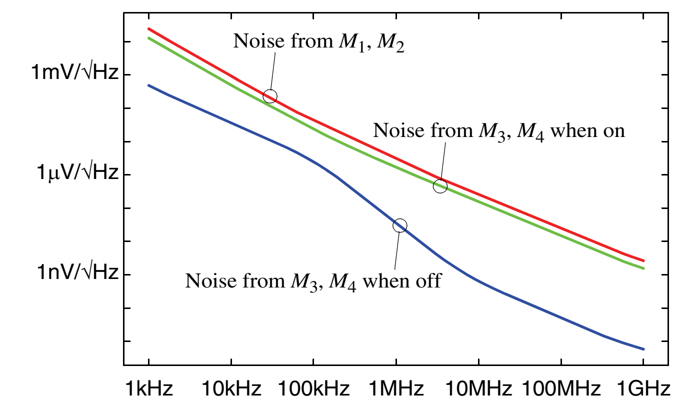
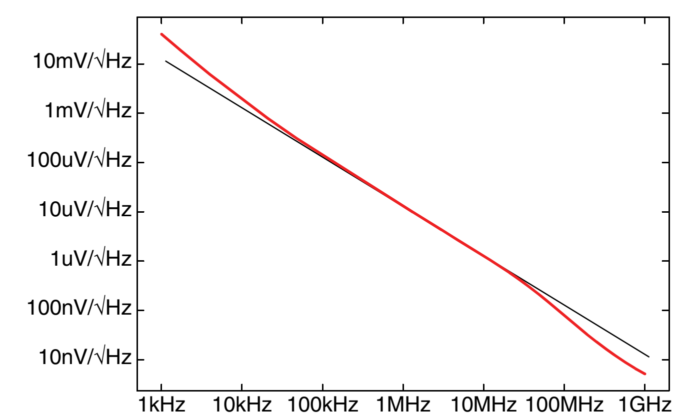


FIGURE 10 *Total output noise when using g_m-based flicker noise models (flkmod=1). The straight thin black line has a slope of 1 and was added to emphasize the flicker noise corner near 10 kHz.*



5.1 If You Have Questions

If you have questions about what you have just read, feel free to post them on the *Forum* section of *The Designer's Guide Community* website. Do so by going to *www.designers-guide.org/Forum*.

Bibliography

[1] Robert E. Collin. Foundations for Microwave Engineering. McGraw-Hill, 1966.

[2] Ken Kundert. Modeling varactors. *www.designers-guide.org/Modeling*.