Using polyphase filters as image attenuators

Using active polyphase filters in receivers with image rejection mixers not only reduces component count, but also provides some unexpected benefits.

By Tom Hornak

A large market exists for low-cost, gigahertz-band radio receivers used for data communications. Low cost demands single-chip implementation with minimum off-chip components. An important goal is to replace any external intermediate frequency (IF) filters with on-chip IF filters while maintaining sufficient image rejection. One possible solution for this challenge is a direct conversion approach—for example, using a zero IF frequency. But direct conversion has many well-known drawbacks, such as DC offset, 1/f noise and local oscillator leak-through. A solution that avoids these problems uses a low-megahertz IF frequency, where on-chip filters can be built within the high-frequency limitations of IC processes[1].

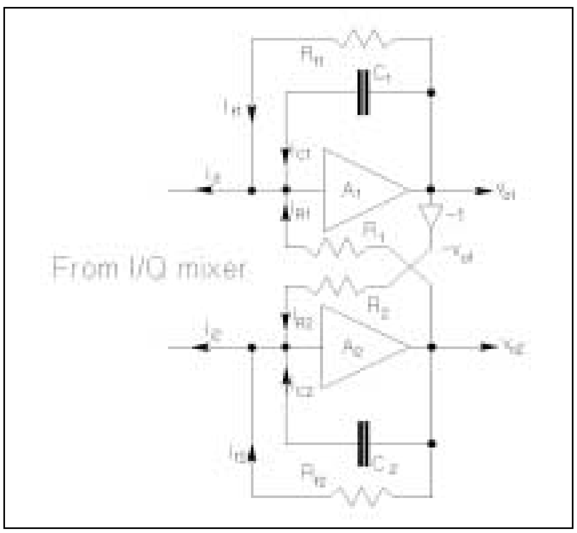

Figure 1. An active polyphase filter stage.

However, a low IF keeps the image frequency so close to the target frequency that suppressing the image in front of the mixer would require an impossibly high Q of the filter preceding the mixer. The solution here is to use an image-rejecting I/Q mixer that delivers two outputs in quadrature to two IF filters. The target signal is then separated from the image signal by the unique phase difference between the two mixer outputs. If the first output's phase lags behind the second output's phase by 90° for the target signal, then the first output's phase leads the second output's phase by 90° for the image signal. The two mixer outputs are usually then filtered by two separate matched IF filters that do not discriminate between the target signal and the image signal. Image rejection is then achieved by an image rejector that shifts the phase of one filter's output by an additional 90° and adds it with the second filter's output. When choosing the proper setup, the two filter outputs from the target signal enter the rejector in phase and add up, while the two filter outputs of the image signal enter the rejector in opposite phase and get subtracted.

Enter the polyphase filter

A better possibility is to replace the two separate filters with one polyphase filter[1]. This technique has three advantages. First, the frequency response of a polyphase filter depends on the phase difference between its two input signals. So, contrary to two separate filters, it has a passband response for the target signal and an attenuating response for the image signal. In low-IF data receivers, the data bandwidth is a significant fraction of the IF filter's center frequency, i.e. the IF filters must have low Q. The frequency response of conventional low-Q bandpass filters is not symmetrical around the passband's center frequency. That distorts the received data's eye diagram.

The second advantage of the polyphase filter is that its bandpass response is symmetrical around the passband's center frequency, independent of its Q. (The polyphase filter keeps the data's eye diagram intact.)

The third advantage of polyphase filters is that, for the same degree of image suppression, the matching of their components is less stringent than the required matching in two separate IF filters and in the subsequent image rejector. This is because of the polyphase filters' close cross-coupling.

Polyphase filters can be entirely passive, built of only resistors and capacitors (see Figure 6 of [1]). An implementation more suitable for monolithic integration is the active polyphase filter [2, 3]. The operation of an active polyphase filter is not obvious from its circuit topology. But analyzing the filter's voltage and current phasors leads to a clearer understanding of the filter's useful properties. The filter's bandpass response to the target, its attenuation of the image and its sensitivity to component mismatch will be examined. All voltage and current symbols appearing in the following sections represent magnitudes. The phase relations are described in the diagrams.

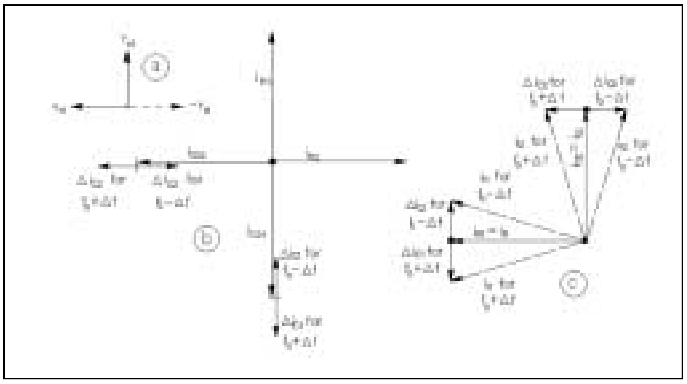

Figure 2. Phasors of the polyphase filter receiving a target signal

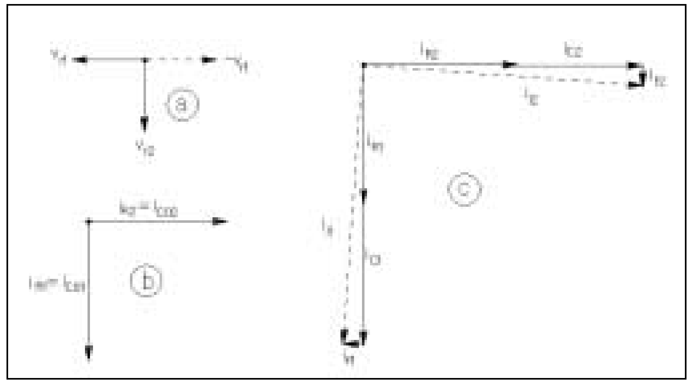

Figure 3. Phasors of the polyphase filter receiving an image signal.

The polyphase filter stage (see Figure1) includes two damped integrators (A, C, R_f), two cross-coupling resistors (R) and one unity-gain inverting amplifier (-1). In an ideal polyphase filter, all components with Suffix 1 exactly match the same component with Suffix 2. In an ideal polyphase filter, the operational amplifiers (A) also have sufficient open-loop gain at the filter's operating frequency band to keep the amplifiers' input voltages within very small fractions of the amplifiers' respective output voltages. Then, the voltage across all resistors and the integrating capacitors (C) is essentially equal to the filter's output voltage. In monolithic IC form, it is preferable to build the filter in full differential mode (and not as shown for clarity in Figure 1). In that case, the filter has four input signals phase-staggered by 90°. The unity-gain inverting amplifier (-1) is implemented simply by crossing two leads.

When measuring a polyphase filter's frequency response, one would normally test the filter's transimpedance Z(f) = v_o/i_i (note how the filter's output voltages v_o vary with frequency f when the input currents i_i are kept fixed). However, understanding this filter's operation is easier when investigating the filter's transadmittance Y(f) = i_i/v_o. Thus, follow how the filter's input currents i_i must be varied with frequency f to keep the filter's output voltages v_o fixed and equal to preset reference voltages v_{r1} and v_{r2}, respectively.

Bandpass property

Figure 2a shows the integrators' fixed output voltage v_{r1} and v_{r2} and the inverted output voltage $-v_{r1}$. Phasor v_{r1} leads v_{r2}. Figure 2b shows current i_R in resistors R and i_{C0} in capacitors C when the filter's input is at its center frequency f_0 . As follows from Figure 1, current i_{R1} is in phase with voltage v_{r2}, and current i_{R2} is in phase with $-v_{r1}$. Capacitor current i_{C01} leads voltage v_{r1} and is in opposite phase with current i_{R1}. Current i_{C02} leads voltage v_{r2} and is in opposite phase with current i_{R2}.

To tune the filter's center frequency to f_0, resistors R are set to $R = 1/(2\pi f_0 C)$. The filter's bandwidth f_b is set by the feedback resistors $Rf = 1/\pi f_b C$. The filter's Q, or the ratio f_0/f_b, is $Q = R_f/(2R)$. Following Figure 1, for any frequency f, the current in resistors R is $i_R = v_r/R$, the current in resistors R_f is $i_f = v_r/R_f$, and the current in the integrating capacitors C is $i_C = 2\pi f C v_r$. At center frequency f_0, the capacitor currents are $i_{C0} = 2\pi f_0 C v_r$, and because $R = 1/(2\pi f_0 C)$, $i_R = i_{C0}$. The filter's input currents i_{i0} are the sum of i_f , i_R and i_{C0}. Thus, at f_0 with i_R and i_{C0} of opposite phase, the filters' input currents i_{i0} are equal to the filter's feedback currents i_f and the output voltages are $v_r = i_{i0}R_f$.

Figure 2c shows the feedback current i_{f1} and i_{f2}, again in phase with their respective drive voltage v_{r1} and v_{r2}, as follows from Figure 1. As stated above, at center frequency, f_0 input currents i_i match the feedback currents i_f. Therefore, in Figure 2c, phasors i_{i0} are identical with phasors i_f.

When $f = f_0$, currents i_{C0} in Figure 2b are equal to i_R and cancel one another. However, with $f \neq f_0$, currents i_C in the integrating capacitors (C) are different from i_{C0}. Thus, with a fixed capacitor voltage $v_c = v_r$, the capacitors demand difference currents $\Delta i_C = i_C - i_{C0} = 2\pi(f - f_0)Cv_r$. When $f = f_0 + \Delta f$ ($\Delta f > 0$), the reactance of capacitors (C) is smaller, thus currents i_C must increase, and currents Δi_C for $f = f_0 + \Delta f$ are in phase with current i_{C0}. On the contrary, when $f = f_0 - \Delta f$, the reactance of capacitor (C) is larger, thus currents Δi_C for $f = f_0 - \Delta f$ are of opposite phase than currents i_{C0}. However, with v_o fixed at v_r, the currents $i_R = v_r/R$ and feedback currents $i_f = v_r/R_f$ are also fixed, independent of frequency f and equal to their magnitude at f_0. Therefore, difference currents Δi_C can come neither from R nor from R_f, but can come only from the filter inputs. Any change Δi_C in capacitor currents i_C due to a deviation from the center frequency f_0 will add an equal component Δi_C to the input currents i_i.

Figure 2c shows the total input currents i_{i1} and i_{i2}, each consisting of a vector sum of current i_f and the currents Δi_C for $f = f_0 + \Delta f$ and $f = f_0 - \Delta f$, respectively. Figure 2c confirms that currents i_i are the lowest at $f = f_0$. Thus, the filter's transimpedance for a target signal is the highest at $f = f_0$ and is equal to:

$$Z_{0t} = \frac{v_r}{i_{i0}} = \frac{v_r}{i_f} = R_f$$

The total input current is:

$$i_i = \sqrt{i_f^2 + \Delta i_c^2} = v_r \sqrt{\left(\frac{1}{R_f^2}\right) + (2\pi C)^2 (f - f_0)^2}$$

With input current i_i being a quadratic function of the frequency difference $f - f_0$, the filter's transadmittance Y(f) = i_i/v_r is symmetrical around center frequency f_0. The same applies to the filter's transimpedance Z(f) = 1/Y(f). As stated in the introduction, this is one of the benefits of polyphase bandpass filters when applied to low IF data receivers.

Image suppressing property

Phasor v_{r1} was chosen, leading v_{r2} in Figure 2, to achieve a bandpass response of the filter. Thus, to ensure an attenuating response, a phasor v_{r2} will now be chosen, leading v_{r1}.

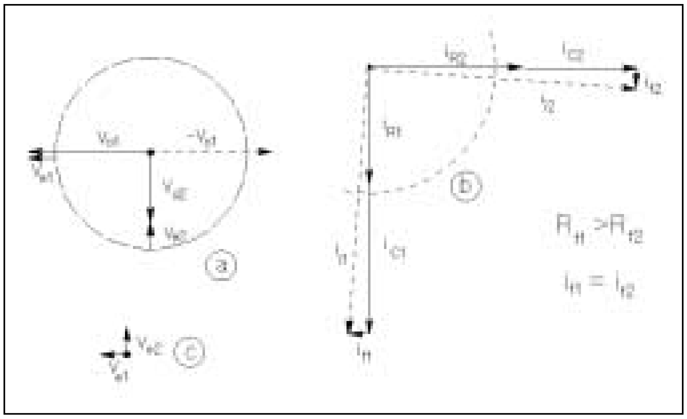

Figure 4. Mismatch of feedback resistors R_f.S.

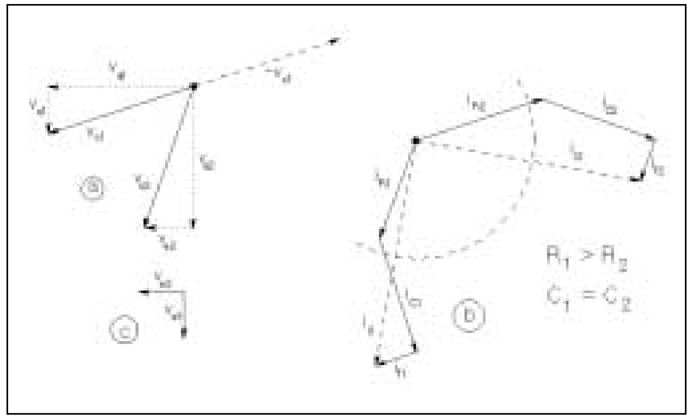

Figure 5. Mismatch of cross-coupling resistors R_1 and R_2.

The situation at frequency f_0 will be followed, which is also the center frequency of the image signal. Figure 3a shows output voltage v_{r1} and v_{r2} and the inverted output voltage $-v_{r1}$. Figure 3b shows the current i_R in resistors (R) and current i_{C0} in capacitors (C). As follows from Figure 1, current i_{R1} is in-phase with voltage v_{r2}, and current i_{R2} is in-phase with $-v_{r1}$. Capacitor currents i_{C01} and i_{C02} lead the respective output voltages v_{r1} and v_{r2}.

Following Figure 1, the current in resistors (R) is $i_R = v_r/R$, and substituting for (R), $i_R = 2\pi f_0 C v_r$. The current in the integrating capacitors (C) is $i_{C0} = 2\pi f_0 C v_r = i_R$. In Figure 2b, the phase of capacitor currents i_{C0} was opposite the phase of currents i_R, therefore they canceled and made no contribution to input current i_{i0}. In Figure 3b, the phase of capacitor current i_C is the same as the phase of current i_R. Therefore, as shown in Figure 3c, their contribution to input current i_{i0} is their sum.

Figure 3c displays feedback current i_{f1} and i_{f2}, again, in phase with their respective drive voltages v_{r1} and v_{r2}. At frequency f_0, the filter's input currents i_{i0} will be the vector sum of feedback currents i_f, i_R and i_{C0}, and with $i_{C0} = i_R$, the vector sum of i_f and $2i_R$. With $i_f = v_r/R_f$ and $i_R = v_r/R$, input currents i_{i0} at frequency f_0 will be:

$$i_{i0} = v_r\sqrt{\left(\frac{1}{R_f^2}\right)+\left(\frac{4}{R^2}\right)}$$

The transimpedance for the image will be:

$$Z_{0i} = \frac{v_r}{i_{i0}} = \frac{1}{\sqrt{\left(\frac{1}{R_f^2}\right)+\left(\frac{4}{R^2}\right)}}$$

Comparing Figs. 2c and 3c, it can be seen that the filter's bandpass response occurs when input current i_{i1} leads input current i_{i2}, while attenuating response occurs when input current i_{i2} leads input current i_{i1}.

The degree of image suppression S_0 at frequency f_0 is equal to the ratio of the filter's transimpedance Z_{0i} for the image signal and the transimpedance Z_{0t} for the target signal. Substituting for $Z_{0t} = R_f$ yields:

$$\frac{Z_{0i}}{Z_{0t}} = S_0 = \frac{1}{\sqrt{1+4\left(\frac{R_f}{R}\right)^2}} = \frac{1}{\sqrt{1+16Q^2}} \approx \frac{1}{4}Q$$

Component mismatch sensitivity

In the previous two sections, it was assumed that the polyphase filter's components with Suffix 1 exactly match the same component with Suffix 2. Now analyze the effect of a mismatch between components in Figure1 when the filter input is an image signal, i.e. i_{i2} leading i_{i1}.

In an ideal polyphase filter, the target and image signal differ at the filter output by the unique phase difference of output voltages v_{o1} and v_{o2}, as is the case for the filter's input currents i_i. In the circuit of Figure 1, a target signal will result in a phasor v_{o1}, leading phasor v_{o2} according to Figure 2a. However, the image signal will cause v_{o2} leading v_{o1} according to Figure 3a. Subsequent polyphase filter stages or an image rejector stage will pass any target signal, but will suppress the image further according to their image attenuation.

Always assume that any component mismatch is evenly distributed between the two members of the mismatched pair, i.e. if the fractional mismatch of a pair is p (e.g. p = 1%), one of the components is off by $p/2$, the other by $-p/2$. For clarity, the mismatches shown in the following figures will be much larger than any occurring in a real integrated circuit.

The phasor diagram of a polyphase filter with mismatched feedback resistors R_f is shown in Figure 4. The output voltages v_{o1} and v_{o2} are in quadrature, however, their magnitudes differ by error components v_{e1} and v_{e2}. Current phasors i_R and i_C in Figure 4b depend again on voltages v_o according to the circuit of Figure 1 and are proportionally mismatched as well. However, the sums $i_{R1} + i_{C1}$ and $i_{R2} + i_{C2}$ and feedback currents i_f are not influenced by the mismatch. When combining error components v_{e1} and v_{e2} (in Figure 4c), it can be seen that v_{e1} leads v_{e2}, i.e. a configuration corresponding to a target signal.

The phasor diagram of a polyphase filter with mismatched cross-coupled resistors $R_1 > R_2$ is shown in Figure 5. The effect of the mismatch is that the phase difference between v_{o1} and v_{o2} is less than 90°. However, their magnitudes remain balanced (see Figure 5a). Current phasors i_R, i_C, and i_f in Figure 5b depend again on voltages v_o according to the circuit of Figure 1. As in Figure 3, the vector sums of i_R, i_C, and i_f are equal to the input currents i_i. Because $R_1 > R_2$, current i_{R1} is smaller than current i_{R2}.

The output phasors v_{o1} and v_{o2}, resulting from mismatch of resistors (R), can be decomposed into quadrature components v_{q1} and v_{q2} and respective error components v_{e1} and v_{e2}, as shown in Figure 5a. The quadrature components represent an image signal (vq2 leads vq1) further attenuated by subsequent image rejecting circuits, if any. However, when error components ve1 and ve2 are again separately joined in Figure 5c, it can be seen that ve1 is

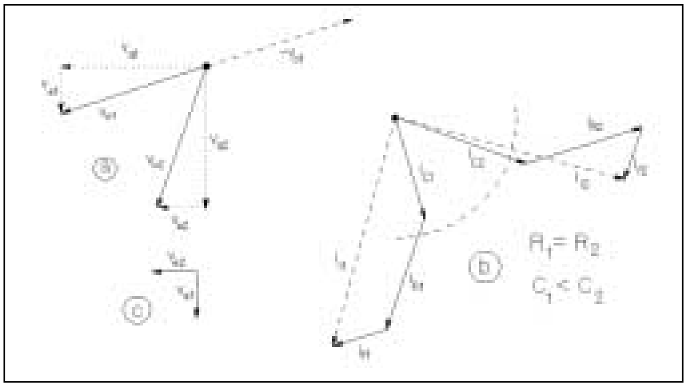

Figure 6. Mismatch of cross-coupling capacitors C_1 and C_2.

leading ve2, again creating a configuration corresponding to a target signal.

When the mismatch is between the integrating capacitors and $C_1 < C_2$, the phase difference between the filter's output voltages is also less than 90° (see Figure 6). When the resistors (R) or the capacitors (C) are mismatched in the opposite sense as described above, the phase difference between the filter's output voltages becomes more than 90° (see Figs. 7 and 8).

Any component mismatch in the polyphase filter results in some error components v_e. Thus, a part of the image signal appears at the filter output as an "image leak" that mimics a target signal. Understandably, any subsequent image-rejecting circuit will pass that leak signal with no attenuation because it cannot distinguish it from a genuine target signal. It is therefore important to develop a quantitative relation between component mismatch and image leak to avoid unpleasant surprises or, on the contrary, to avoid excessive component matching that costs chip area and power dissipation.

To find the relation between mismatch and leak, one must first assume that the filter is not mismatched and that it is driven by an image signal. Its outputs will be $v_{r1} = v_{r2}$, as in Figure 3. Next, mismatch a component pair, but keep the filter output fixed to v_{r1} and v_{r2} and the filter input voltage at zero. The mismatch will cause an error current of i_{e1} in one, and of i_{e2} in the other member of the pair. Finally, assume that a mismatch-free replica of the analyzed filter has error currents i_{e1} and i_{e2} as its input currents i_{i1} and i_{i2}. The magnitude of the replica filter's output will be a good approximation of error components v_{e1} and v_{e2} in the mismatched filters of Figs. 4 and 5.

To calculate error currents i_e when the mismatched pair is the feedback resistors, use the following formulas:

$$R_{f1} = R_f(1 + p/2)$$

and

$$R_{f2} = R_f(1 - p/2).$$

With p << 1:

$$i_{e1} = v_{r1}\left(\frac{1}{R_{f1}} - \frac{1}{R_f}\right) \approx -\frac{v_{r1}\,p}{2R_f}$$

and

$$i_{e2} = v_{r2}\left(\frac{1}{R_{f2}} - \frac{1}{R_f}\right) \approx +\frac{v_{r2}\,p}{2R_f}$$

It can be seen that i_{e1} is of opposite phase to v_{r1}, while i_{e2} is in phase with v_{r2}. Because phasors v_{r1} and v_{r2} represent an image signal (see Figure 3), the constellation of currents i_{e1} and i_{e2} must be that of a target signal. It has been found that the transimpedance for a target signal is $Z_{0t} = R_f$. So, when driving the replica filter with i_{e1} and i_{e2}, its outputs will be $v_e = i_eR_f = v_rp/2$. When p << 1 and the mismatched filter's input is an image signal, its feedback current i_f is in close quadrature with input current i_i, similar to Figure 3. Input current i_i is then close to $i_i = i_R + i_C$ and is almost equally divided between i_R and i_C.

Thus, with good approximation, one can write:
$v_r = i_iR/2$.

And, furthermore, for a mismatch p in R_f:

$v_{ef} = i_iRp/4$.

When the same procedure is applied with the mismatched pair using the cross-coupled resistors, the result is:

$v_{eR} = i_iRfp/4$.

Finally, the same applies when the mismatched pair is the integrating capacitors C_1 and C_2.

To assess the significance of the image leak v_e, the filter's transimpedance Z_{0t} must be compared for a target signal with its transimpedance Z_{0p} for an image signal with a mismatch in R_f, R or C, respectively. It is known that $Z_{0t} = R_f$. From this:

$Z_{pf} = v_{ef}/i_i = Rp/4$ for a mismatch of feedback resistors R_f, and:

$Z_{pR} = v_{eR}/i_i = Rfp/4$ for a mismatch of cross-coupled resistors (R) or capacitors (C).

One important ratio is $Z_{ipf}/_{ZipR} = R/R_f = 1/2Q$. It means that the matching of

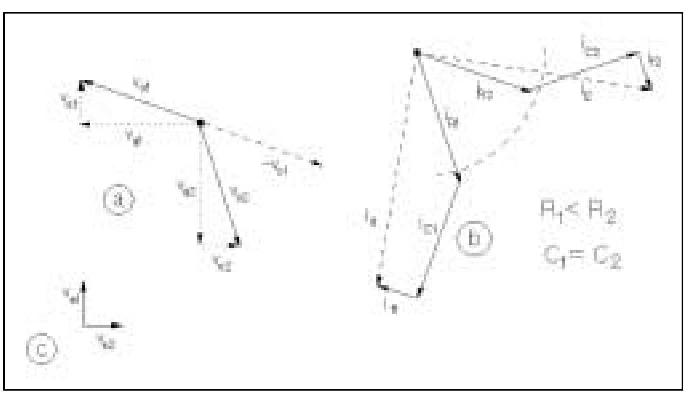

Figure 7. Mismatch of cross-coupling resistors opposite to Figure. 5.

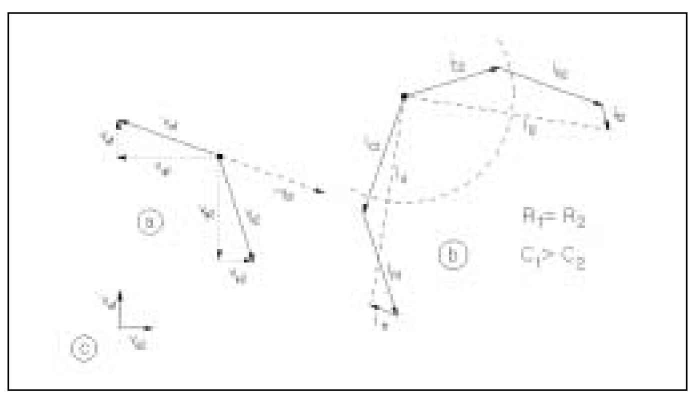

Figure 8. Mismatch of cross-coupling capacitors opposite to Figure 6.

R_f can be relaxed 2Q-times over the matching of R or C to cause the same image leak. Another important ratio is $Z_{pR}/Z_{0t} = p/4$. This says that if, for example, the ratio of image signal to target signal is 1000:1 (60 dB), to keep the image leak smaller than the target signal, the mismatch *p* of (R) or (C) can be as high as 0.4%. This is one advantage of the polyphase filter over two separate IF filters, where, in the same case, a mismatch of no worse than 0.1% is required.

The assessment

The operation of an active polyphase filter when used for image attenuation in low IF data receivers has been clearly visualized by a phasor analysis of the filter's voltages and currents. The influence of the filter's component mismatch on its image suppression performance has been quantitatively analyzed. It has been shown that the image-attenuating performance of a polyphase filter is superior to two separate IF filters.

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