A. Single-Stage MOSFET Amplifiers

1. Introduction

-In this document, we will derive the expressions for the small-signal parameters (Rin, Rout, and Av) for the 4 basic types of MOSFET single-stage amplifiers.

- Our focus will on deriving results that can be *applied* to other single-stage amplifiers to obtain expressions and values for the key parameters of those amplifiers.

We will assume the following values for the DC and small-signal parameters for each transistor:

$ID := 1mA$ $\quad gm := 0.5\frac{mA}{V}$ $\quad Rsig := 2k\Omega$ $\quad RG := 2k\Omega$

$ro := 500k\Omega$ $\quad gmb := 50\frac{\mu A}{V}$ $\quad RD := 50k\Omega$ $\quad RS := 2k\Omega$

General approach and notes:

a. For each amplifier type, we will derive the entire expression for Rin, Rout, and Av.

b. In practice, we will apply these results to specific amplifiers to determine desired expressions.

c. In practice, the expressions can (and should) be simplified, if possible. Care should be to apply the proper simplifications.

d. The biasing details of the amplifiers are not shown.

2. Common-Source (CS) Amplifier:

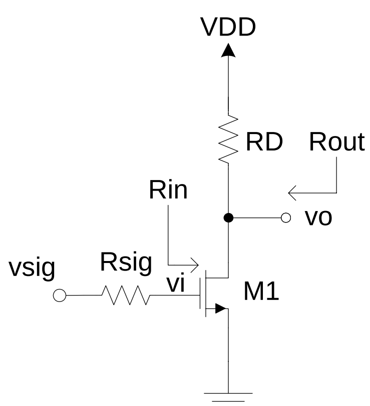


Figure 1. A common-source amplifier.

Rsig is the total the total resistance seen by M1 looking out from the gate. In practice, it may be a "source resistor" or the output resistance of a "preceding stage".

RD is the total drain resistance, that is total the total resistance seen by M1 looking out from the drain. In practice, it may also contain a "load resistance", RL, connected from "vo" to ground. Also RL may be an actual load or the input resistance from a "subsequent stage".

Rsig is *never* included in the calculation for Rin. If RL is present, it is *never* included in the calculation of Rout. They are not considered part of the amplifier.

vi is the signal at the "input" terminal of the MOST.

a. The derivation of Av, Rin, and Rout:

The small-signal equivalent circuit is:

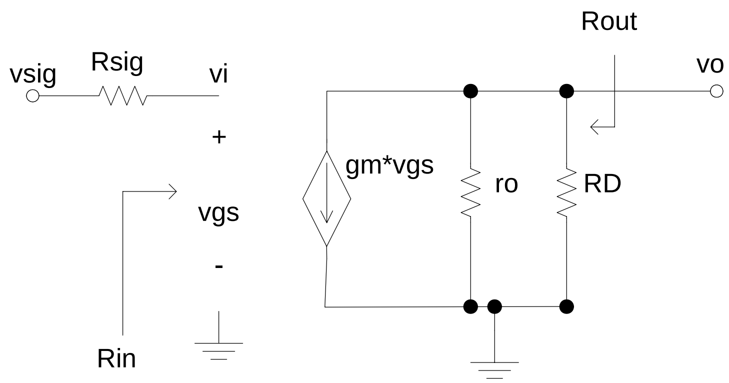


Figure 2. The small-signal equivalent circuit for the common-source amplifier

The expressions for the parameters can be determined by inspection:

$$\boxed{Rin = \infty}$$

$$\boxed{Rout = ro \| RD}$$

$$\boxed{\frac{vo}{vi} = -gm \cdot (ro \| RD)}$$

$$\boxed{Av = \frac{vo}{vsig} = \frac{vo}{vi} = -gm \cdot (ro \| RD)}$$

Note: Depending on the topology, vo/vsig is not always equal to vo/vi.

Substituting in the values for the small-signal parameters and finding the values for Rin, Rout, and Av gives:

$Rin = \infty$ $\qquad$ $Rout = 45.455\,k\Omega$ $\qquad$ $Av = -22.727\frac{V}{V}$

b. Simplifications (note there is no body effect in the amplfier):

(1) The condition for neglecting the effects of CLM is: ro >> RD

Then, $\boxed{Rin = \infty}$ $\boxed{Rout = RD}$ $\boxed{Av = -gm \cdot RD}$ **....1st order equations for the CS amplifier**

3. Common-Source (CS/SR) with Source Resistor (source degeneration)

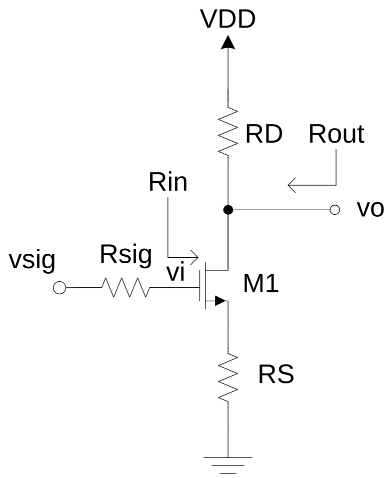


Figure 3. A common-source with source resistor amplifier

Rsig is the total the total resistance seen by M1 looking out from the gate. In practice, it may be a "source resistor" or the output resistance of a "preceding stage".

RD is the total drain resistance, that is total the total resistance seen by M1 looking out from the drain. In practice, it may also contain a "load resistance", RL, connected from "vo" to ground. Also RL may be an actual load or the input resistance from a "subsequent stage".

RS is the total the total resistance seen by M1 looking out from the source.

Rsig is *never* included in the calculation for Rin. If RL is present, it is *never* included in the calculation of Rout. They are not considered part of the amplifier.

vi is the signal at the "input" terminal of the MOST.

The small-signal equivalent circuit is:

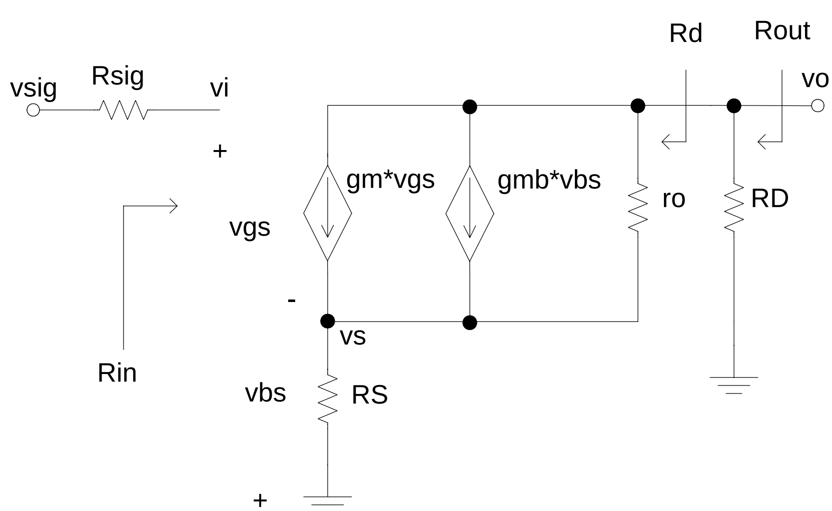


Figure 4. The small-signal equivalent circuit for the common-source with source resistor amplifier

a. Rin **=** ∞ **by inspection.**

b. Rout:

To find Rout, we will first find Rd, the small-signal resistance looking into the drain of M1. This situation comes up so often, it is worth generalizing this first, and then applying it to the CS/SR amplifier.

Derivation for the small signal resistance seen looking into the drain of a MOSFET, Rd.

A typical DC MOSFET circuit is shown as follows:

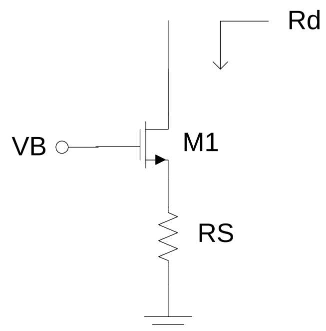
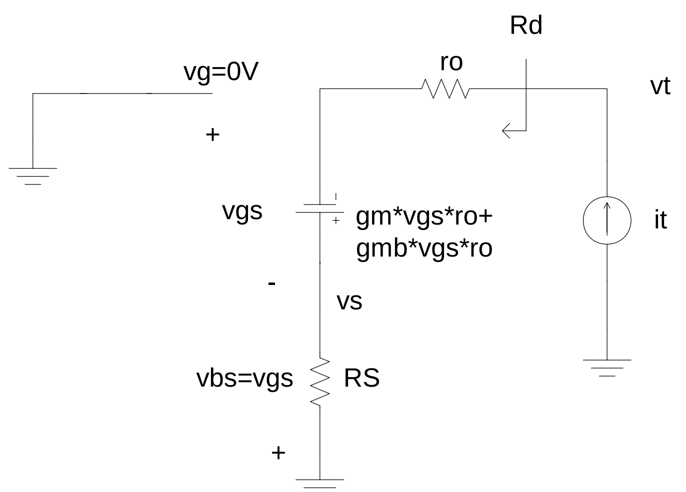


Figure 5. DC MOSFET circuit with Rd, the small-signal resistance looking into the drain, depicted.

The drain circuitry is not shown. RS may be either a resistor, or some equivalent resistance seen looking *from* the source of M1 to ground, often realized by a single or multiple transistor circuit.

VB is a DC biasing voltage.

The small-signal equivalent circuit for determing Rd is:



We note from the small-signal equivalent circuit that our derivation may be used *whenever the gate is at signal ground.*

Figure 6. The small-signal equivalent circuit for determing Rd.

Since vbs = vgs, the current sources are combined. Then, the combination of the current sources an ro are Thevenized, resulting in a voltage source in series with ro.

By inspection:

$$vt = it \cdot ro - (gm + gmb) \cdot vgs \cdot ro + it \cdot RS = it \cdot ro - (gm + gmb) \cdot vs \cdot ro + it \cdot RS$$

$$vt = it \cdot ro - (gm + gmb) \cdot (-it \cdot RS) \cdot ro + it \cdot RS$$

Thus $$\boxed{Rd = \frac{vt}{it} = ro + RS + (gm + gmb) \cdot ro \cdot RS}$$

Common simplifications: (IMPORTANT: This comes up often)

1) gm >> gmb

2) gm*ro >> 1

3) RS $\approx$ ro (RS may be realized by a single transistor)

1) gm >> gmb

We note that often in practice, gm >> gmb and can be determined by inspection. Recall that

$gmb = X \cdot gm$ **where** X **is between 0.1 and 0.3.**

2) gm*ro >> 1

Note that $$gm \cdot ro = \frac{2 \cdot ID}{Vov} \cdot \frac{1}{\lambda \cdot ID} = \frac{2}{Vov \cdot \lambda}$$

For typical values of Vov and λ, $Vov := 250mV$ **and** $\lambda := 0.01 \cdot V^{-1}$

$$gmro := \frac{2}{Vov \cdot \lambda} \quad gmro = 800$$ **....we see that this condition is generally true.**

Other forms of this approximation are: a) gm >> 1/ro and b) ro >> 1/gm.

3) RS $\approx$ ro. This comes up often, but must be evaluated on a case-by-case basis.

Going back to the expression for Rd, and invoking simplifications 1, 2, and 3:

$$Rd = ro + RS + (gm + gmb) \cdot ro \cdot RS$$

$Rd \approx (ro + RS + gm \cdot ro \cdot RS)$ **gm >> gmb**

$Rd \approx [ro + RS \cdot (1 + gm \cdot ro)]$

$Rd \approx (ro + RS \cdot gm \cdot ro)$ **gmro >> 1**

$Rd \approx [ro \cdot (1 + gm \cdot RS)]$

$Rd \approx (gm \cdot ro \cdot RS)$ **gmro >> 1 and remembering RS ≈ ro**

Refering to Fig. (3) and Fig. (4) we see that the gate of M1 is at signal ground since vsig is grounded for the Rout analysis and there is no voltage drop across Rsig since ig=0. Thus, the results of the derivation of Rd apply, and we have:

Thus $Rd = ro + RD + (gm + gmb) \cdot ro \cdot RS$

and $Rout = RD \parallel [ro + RS + (gm + gmb) \cdot ro \cdot RS]$

c. Av

To find Av, we first find Gm. Then, Av = -Gm*Rout. The reason for this approach, is that it is the approach that uses the fewest and easiest calculations.

Remember, to find Gm, we short the output and find "io" the current through the short. Then, Gm is equal to io/vsig. It is conventional to define "io" going "into" the amplifier.

Shorting the output shorts out RD, thus, we may remove it from the circuit. The resulting small-signal equivalent circuit is:

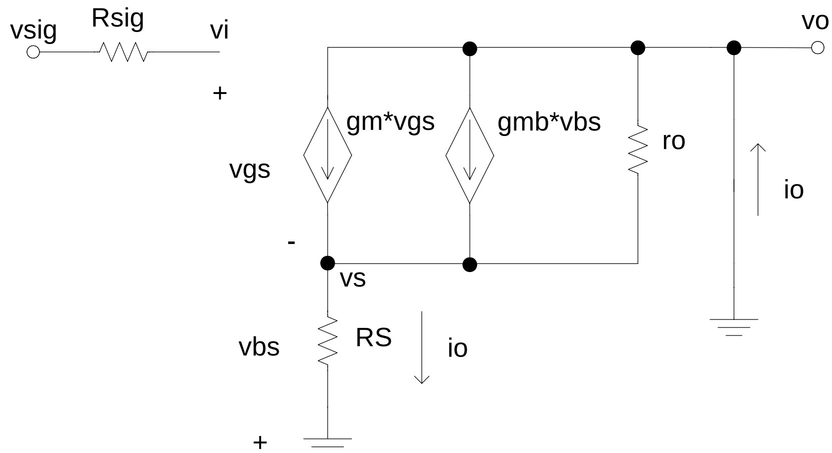


Figure 7. The small-signal equivalent circuit for determing Av for the common-source with source resistor amplifier.

Note that "io" must flow through RS. The current through RS is determined by recogizing that since both ro and RS are grounded, and in parallel, the current from the current sources divides between the two resistors in accordance to the current divider rule.

Thus

$$io = \frac{ro}{ro + RS} \cdot (gm \cdot vgs + gmb \cdot vbs)$$

Since $vbs = -io \cdot RS$ **and** $vgs = vi - vs = vi - io \cdot RS$

$$io = \frac{ro}{ro + RS} \cdot [gm \cdot (vi - io \cdot RS) + gmb \cdot (-io \cdot RS)]$$

Solving for io and then io/vi, results in

$$\frac{io}{vi} = Gm = \frac{gm \cdot ro}{ro + RS + (gm + gmb) \cdot ro \cdot RS} = \frac{io}{vsig}$$

Thus,

$$Av = -Gm \cdot Rout = -\frac{gm \cdot ro}{ro + RS + (gm + gmb) \cdot ro \cdot RS} \cdot [[RD \parallel [ro + RS + (gm + gmb) \cdot ro \cdot RS]]]$$

$$Av = -\frac{gm \cdot ro}{ro + RS + (gm + gmb) \cdot ro \cdot RS} \cdot \frac{RD \cdot [ro + RS + (gm + gmb) \cdot ro \cdot RS]}{RD + ro + RS + (gm + gmb) \cdot ro \cdot RS}$$

$$Av = -\frac{gm \cdot ro \cdot RD}{RD + ro + RS + (gm + gmb) \cdot ro \cdot RS} = \frac{-gm \cdot RD}{1 + \left(gm + gmb + \frac{1}{ro}\right) \cdot RS + \frac{RD}{ro}}$$

In summary, for the common-source with source resistor:

$$Rin = \infty$$

$$Rout = RD \parallel [ro + RS + (gm + gmb) \cdot ro \cdot RS]$$

$$Av = \frac{-gm \cdot RD}{1 + \left(gm + gmb + \frac{1}{ro}\right) \cdot RS + \frac{RD}{ro}}$$

Substituting in our numbers gives:

$$\text{Rin} = \infty \qquad \text{Rout} = 45.455\,\text{k}\Omega \qquad \text{Av} = -11.882\,\frac{\text{V}}{\text{V}}$$

d. Simplifications:

i) The conditions for neglecting the body effect are: gm >> gmb

Then

$$\text{Rin} = \infty$$

$$\text{Rout} = \text{RD} \parallel (\text{ro} + \text{RS} + \text{gm}\cdot\text{ro}\cdot\text{RS})$$

$$\text{Av} = \frac{-\text{gm}\cdot\text{RD}}{1 + \left(\text{gm} + \frac{1}{\text{ro}}\right)\cdot\text{RS} + \frac{\text{RD}}{\text{ro}}}$$

ii) A "primary" condition for neglection CLM is (assuming the body effect is neglected): ro >> 1/gm, RD

Then

$$\boxed{\text{Rin} = \infty}$$

$$\boxed{\text{Rout} = \text{RD}}$$

$$\boxed{\text{Av} = \frac{-\text{gm}\cdot\text{RD}}{1 + \text{gm}\cdot\text{RS}}}$$

....1st order equations for the CS/SR amplifier

iii) It is also possible to neglect CLM but include the Body Effect.

4. The Common Gate (CG) Amplifier

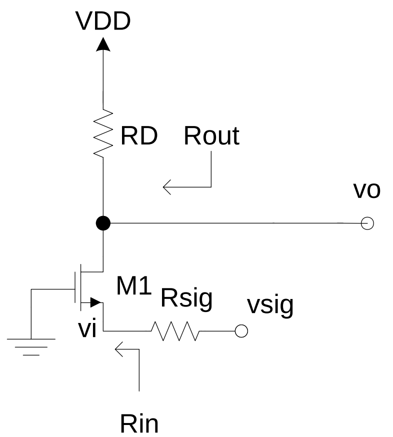


Figure 8. A common-gate amplifier

RD is the total drain resistance, that is total the total resistance seen by M1 looking out from the drain. In practice, it may also contain a "load resistance", RL, connected from "vo" to ground. Also RL may be an actual load or the input resistance from a "subsequent stage".

Rsig is the total the total resistance seen by M1 looking out from the source. It may be part of the amplifier or it may be part of the driving source vsig. It may also consist of a component that is part of the amplifier and one from the source. It may also represent the output resistance from a "preceding stage".

Rsig is *not* included in the Rin calculation, unless it is part of the amplifier. If RL is applied, it is *never* included in the Rout calculation, since it is ususally considered to be not part of the amplifier.

For this example, we assume that it is not part of the amplifier, but is part of the "driving source resistance".

vi is the signal at the "input" terminal of the MOST.

a. Rin and vo/vi:

Due to the relatively small input impedance of the common-gate amplifier, in order to find the gain of the circuit we first find vo/vi and then Rin. Then,

$$\frac{vo}{vsig} = \frac{vi}{vsig} \cdot \frac{vo}{vi} = \frac{Rin}{Rin + RS} \cdot \frac{vo}{vi}$$

The small-signal equivalent circuit is:

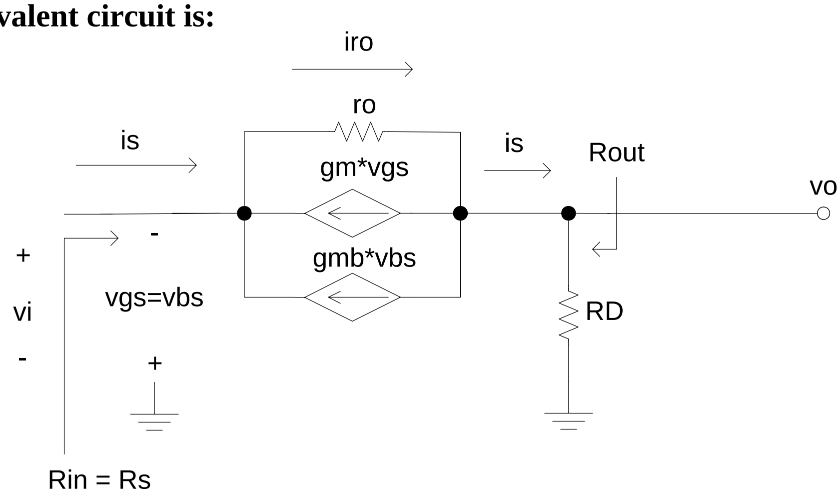


Figure 9. Small-signal equivalent circuit for the common-gate amplifier (wo Rsig and vsig)

To detemine the input resistance for the amplifier, we need to find the small-signal resistance seen looking into the source, Rs. This situation comes up so often, it is worth generalizing this first, and then applying it to the CG amplifier.

Derivation for the small signal resistance seen looking into the source of a MOSFET, Rs.

A typical DC MOSFET circuit is shown as follows:

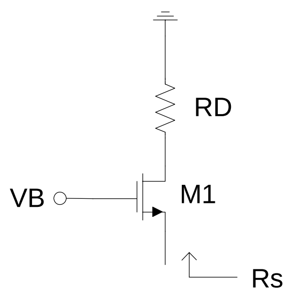
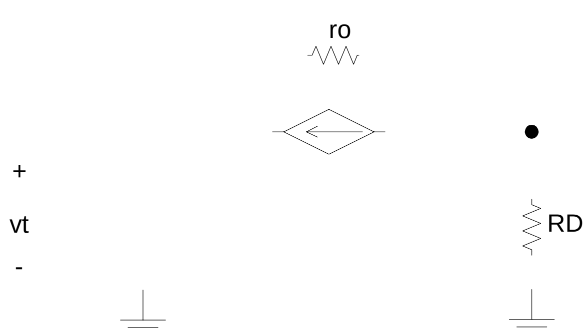


Figure 10. DC MOSFET circuit with Rs, the small-signal resistance looking into the drain, depicted.

The drain circuitry is not shown. RD may be either a resistor, or some equivalent resistance seen looking *from* the drain of M1 to ground, often realized by a single or multiple transistor circuit.

VB is a DC biasing voltage.

The small-signal equivalent circuit for determing Rs is:



We note from the small-signal equivalent circuit that our derivation may be used *whenever the gate is at signal ground.*

Figure 11. Small-signal equivalent circuit used to determine Rs.

We see that

$$vt = iro \cdot ro + it \cdot RD = (it + gm \cdot -vt + gmb \cdot -vt) \cdot ro + it \cdot RD$$

Solving for vs, find:

$$vt = \frac{RD + ro}{1 + (gm + gmb)\cdot ro}\cdot it$$

Thus $$Rs = \frac{vs}{is} = \frac{RD + ro}{1 + (gm + gmb)\cdot ro} = \frac{1 + \frac{RD}{ro}}{gm + gmb + \frac{1}{ro}}$$

$$\boxed{Rs = (1/gm \parallel 1/gmb \parallel ro)\cdot\left(1 + \frac{RD}{ro}\right)}$$

If gm >> gmb and gm*ro >> 1

$$Rs = 1/gm\cdot\left(1 + \frac{RD}{ro}\right)$$

If ro >> RD

$$\boxed{Rs = 1/gm}$$ **This occurs often!**

Applying the results of Rs to the CG amplifier,

Thus

$$Rin = \frac{vi}{is} = (1/gm \parallel 1/gmb \parallel ro)\cdot\left(1 + \frac{RD}{ro}\right)$$

Now, referring to Fig. (9),

$$vo = is\cdot RD = RD\cdot\frac{vi}{Rin} = \frac{RD}{(1/gm \parallel 1/gmb \parallel ro)\cdot\left(1 + \frac{RD}{ro}\right)}\cdot vi = \left(gm + gmb + \frac{1}{ro}\right)\cdot(ro\|RD)\cdot vi$$

So

$$\frac{vo}{vi} = \left(gm + gmb + \frac{1}{ro}\right) \cdot ro||RD$$

And

$$\boxed{Av = \frac{Rin}{Rin + RS} \cdot \left[\left(gm + gmb + \frac{1}{ro}\right) \cdot ro||RD\right]}$$

b. Rout:

Rout is the small-signal equivalent circuit seen looking into the drain, Rd, in parallel with RD.

Thus, $Rout = RD \parallel [ro + Rsig + (gm + gmb) \cdot ro \cdot Rsig]$

In summary, for the common-gate amplifier:

$$\boxed{Rin = (1/gm \parallel 1/gmb \parallel ro) \cdot \left(1 + \frac{RD}{ro}\right)}$$

$$\boxed{Rout = RD \parallel [ro + Rsig + (gm + gmb) \cdot ro \cdot Rsig]}$$

$$\boxed{\frac{vo}{vi} = \left(gm + gmb + \frac{1}{ro}\right) \cdot ro||RD}$$

$$\boxed{Av = \frac{vi}{vsig} \cdot \frac{vo}{vi} = \frac{Rin}{Rin + RS} \cdot \left[\left(gm + gmb + \frac{1}{ro}\right) \cdot ro||RD\right]}$$

Plugging in our numbers:

$Rin = 1.993\,k\Omega$ $\quad$ $Rout = 47.731\,k\Omega$ $\quad$ $Av = 12.523\,\frac{V}{V}$

c. Simplifications

The main simplifications are:

To neglected the body effect: **gm >> gmb**

To neglect the effects of CLM: **ro >> 1/gm and ro >> RD.**

Clearly, there are many variations of this. For example, in the expression for Rin, we may have ro >> 1/gm, but not ro >> RD.

Applying these simplifications, yields:

$$\mathrm{Rin} = \frac{1}{\mathrm{gm}} \qquad \mathrm{Rout} = \mathrm{RD} \qquad \frac{\mathrm{vo}}{\mathrm{vi}} = \mathrm{gm}\cdot\mathrm{RD}$$

....1st order equations for the CG amplifier.

$$\mathrm{Av} = \frac{\mathrm{vi}}{\mathrm{vsig}}\,\frac{\mathrm{vo}}{\mathrm{vi}} = \frac{\mathrm{Rin}}{\mathrm{Rin}+\mathrm{RS}}\cdot\mathrm{gm}\cdot\mathrm{RD} = \frac{\frac{1}{\mathrm{gm}}}{\frac{1}{\mathrm{gm}}+\mathrm{RS}}\cdot\mathrm{gm}\cdot\mathrm{RD} = \frac{\mathrm{gm}\cdot\mathrm{RD}}{1+\mathrm{gm}\cdot\mathrm{RS}}$$

5. Common-Drain (CD) Amplifier (Source Follower (SF))

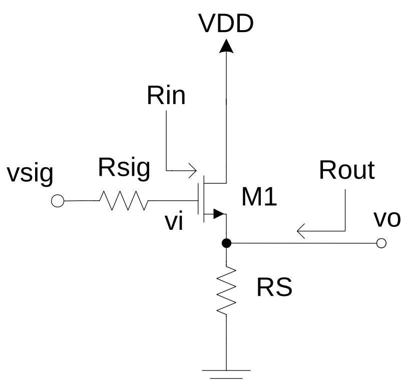


Figure 12. A common-drain amplifier

Rsig is the total the total resistance seen by M1 looking out from the gate. In practice, it may be a "source resistor" or the output resistance of a "preceding stage".

RS is the total the total resistance seen by M1 looking out from the source. In practice, it may also contain a "load resistance", RL, connected from "vo" to ground. RL may be an actual load or the input resistance from a "subsequent stage".

Rsig is *never* included in the Rin calculation, unless it is part of the amplifier. If RL is applied, it is *never* included in the Rout calculation, since it is ususally considered to be not part of the amplifier.

vi is the signal at the "input" terminal of the MOST.

a. vo/vsig and Rin:

Rsig is neglected since the current flowing through it is 0A. The small-signal equivalent circuit is:

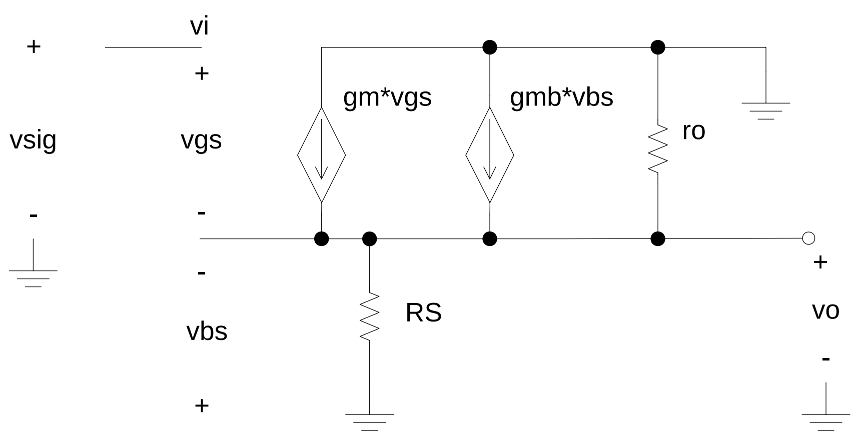


Figure 13. Small-signal equivalent circuit for the Common drain amplifier

We need to simplify this circuit. Note that the controlling voltage (vbs) of the source gmb*vbs is for all practical purposes, directly across itself.

Consider the following situation:

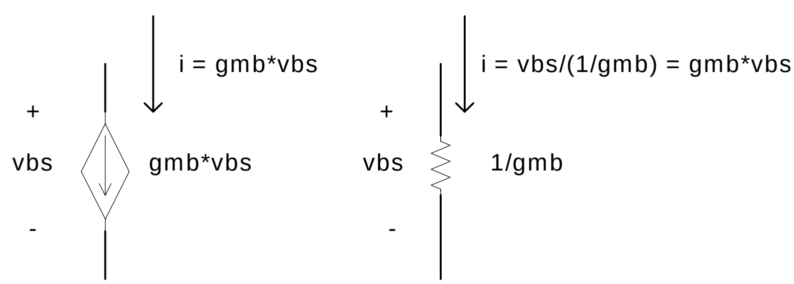


Figure 14. Simplifications for the common-drain amplifier

Applying a voltage of vbs across this controlled source results in a current of gmb*vbs. Applying a voltage of vbs across a resistor of value 1/gmb also results in a current of gmb*vbs. Thus, the circuits are equivalent and the voltage controlled current source may be modeled as a resistor when the controlling voltage is applied across the device with the polarity shown. This comes up often and is worth making a note about.

The small-signal equivalent circuit may be simplified to:

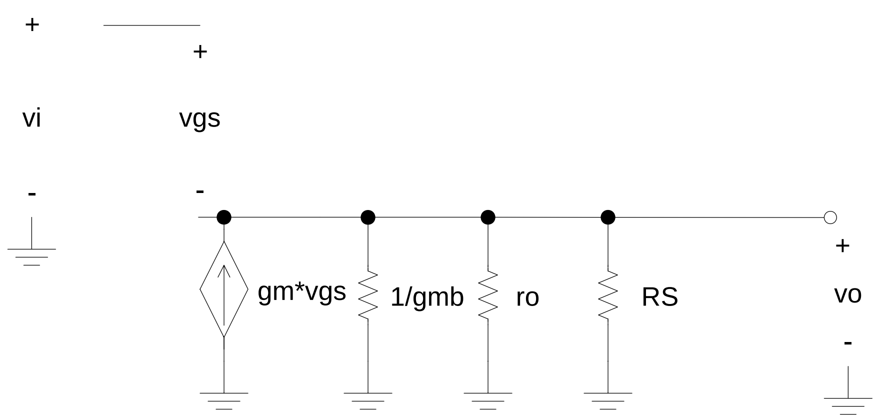


Figure 15. Simplified small-signal equivalent circuit for the common-drain amplifier

Let

$$Rx = 1/gmb||ro||RS$$

Then

$$vo = gm \cdot vgs \cdot Rx = gm \cdot (vi - vo) \cdot Rx$$

Solving for vo/vg, find

$$\frac{vo}{vi} = \frac{vo}{vi} = \frac{gm \cdot (1/gmb||ro||RS)}{1 + gm \cdot (1/gmb||ro||RS)} = Av = \frac{gm \cdot RS}{1 + \left(gm + gmb + \frac{1}{ro}\right) \cdot RS}$$

After some algebra.

By inpection.

$$Rin = \infty$$

b. Rout:

Rout is Rs in parallel with RS.

$$Rs = (1/gm \;||\; 1/gmb \;||\; ro) \cdot \left(1 + \frac{0}{ro}\right)$$

since RD=0Ω

$$Rs = (1/gm \;||\; 1/gmb \;||\; ro)$$

Thus, $Rout = 1/gm||1/gmb||ro||RS$

In summary: $Rin = \infty$ $Rout = 1/gm||1/gmb||ro||RS$ $Av = \frac{gm \cdot RS}{1 + \left(gm + gmb + \frac{1}{ro}\right) \cdot RS}$

Plugging in our numbers, find:

$Rin = \infty$ $Rout = 950.57\,\Omega$ $Av = 0.475\,\frac{V}{V}$

c. Simplifications:

i. To neglect the body effect **1/gmb >> RS, or gmb << 1/RS**

ii. To neglect the effects of CLM **ro >> RS**

Of course, there are several other variations. This yields:

$Rin = \infty$ $Rout = 1/gm||RS$ $Av = \frac{gm \cdot RS}{1 + gm \cdot RS}$

....1st order equations for the CD amplifier.

Often, gmRS >> 1, and Av = 1 V/V.

6. Applying the results

When applying the results, some general guidelines are:

1. Identify the amplifier type.

2. Apply the desired expression. Simplify when possible.

7. Remember, you should be able to *derive* these results as well as *apply* these results.