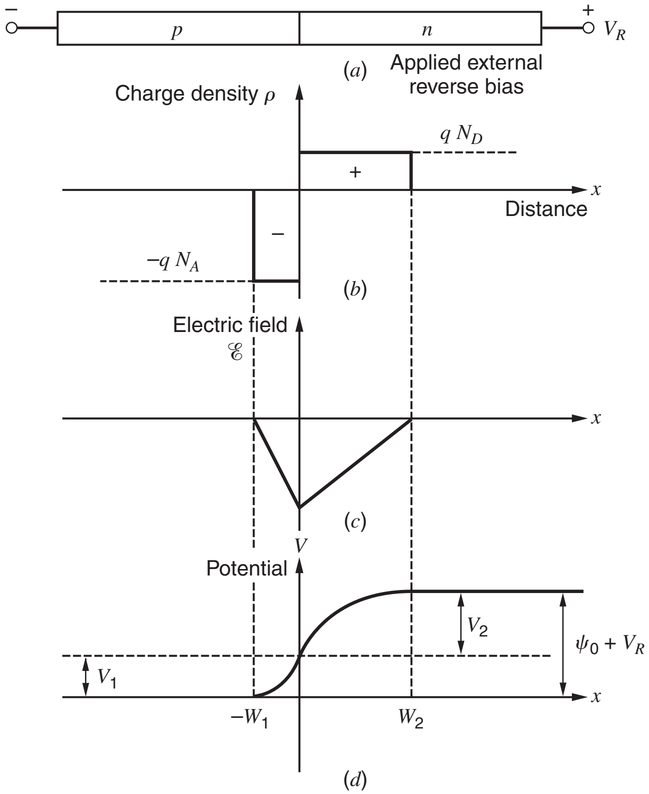

Figure 1.1 The abrupt junction under reverse bias V_R. (*a*) Schematic. (*b*) Charge density. (*c*) Electric field. (*d*) Electrostatic potential.

region. It is assumed that the edges of the depletion region are sharply defined as shown in Fig. 1.1, and this is a good approximation in most cases.

For zero applied bias, there exists a voltage ψ_0 across the junction called the *built-in potential*. This potential opposes the diffusion of mobile holes and electrons across the junction in equilibrium and has a value[1]

$$\psi_0 = V_T \ln \frac{N_A N_D}{n_i^2} \tag{1.1}$$

where

$$V_T = \frac{kT}{q} \simeq 26 \text{ mV} \quad \text{at} \quad 300^\circ\text{K}$$

the quantity n_i is the intrinsic carrier concentration in a pure sample of the semiconductor and $n_i \simeq 1.5 \times 10^{10} \text{cm}^{-3}$ at 300°K for silicon.

In Fig. 1.1 the built-in potential is augmented by the applied reverse bias, V_R, and the total voltage across the junction is $(\psi_0 + V_R)$. If the depletion region penetrates a distance W_1 into the p-type region and W_2 into the n-type region, then we require

$$W_1 N_A = W_2 N_D \tag{1.2}$$

because the total charge per unit area on either side of the junction must be equal in magnitude but opposite in sign.

Poisson's equation in one dimension requires that

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon} = \frac{qN_A}{\epsilon} \quad \text{for} \quad -W_1 < x < 0 \tag{1.3}$$

where ρ is the charge density, q is the electron charge (1.6×10^{-19} coulomb), and ϵ is the permittivity of the silicon (1.04×10^{-12} farad/cm). The permittivity is often expressed as

$$\epsilon = K_S \epsilon_0 \tag{1.4}$$

where K_S is the dielectric constant of silicon and ϵ_0 is the permittivity of free space (8.86×10^{-14} F/cm). Integration of (1.3) gives

$$\frac{dV}{dx} = \frac{qN_A}{\epsilon}x + C_1 \tag{1.5}$$

where C_1 is a constant. However, the electric field $\mathscr{E}$ is given by

$$\mathscr{E} = -\frac{dV}{dx} = -\left(\frac{qN_A}{\epsilon}x + C_1\right) \tag{1.6}$$

Since there is zero electric field outside the depletion region, a boundary condition is

$$\mathscr{E} = 0 \quad \text{for} \quad x = -W_1$$

and use of this condition in (1.6) gives

$$\mathscr{E} = -\frac{qN_A}{\epsilon}(x + W_1) = -\frac{dV}{dx} \quad \text{for} \quad -W_1 < x < 0 \tag{1.7}$$

Thus the dipole of charge existing at the junction gives rise to an electric field that varies linearly with distance.

Integration of (1.7) gives

$$V = \frac{qN_A}{\epsilon}\left(\frac{x^2}{2} + W_1 x\right) + C_2 \tag{1.8}$$

If the zero for potential is arbitrarily taken to be the potential of the neutral p-type region, then a second boundary condition is

$$V = 0 \quad \text{for} \quad x = -W_1$$

and use of this in (1.8) gives

$$V = \frac{qN_A}{\epsilon}\left(\frac{x^2}{2} + W_1 x + \frac{W_1^2}{2}\right) \quad \text{for} \quad -W_1 < x < 0 \tag{1.9}$$

At $x = 0$, we define $V = V_1$, and then (1.9) gives

$$V_1 = \frac{qN_A}{\epsilon}\frac{W_1^2}{2} \tag{1.10}$$

If the potential difference from $x = 0$ to $x = W_2$ is V_2, then it follows that

$$V_2 = \frac{qN_D}{\epsilon}\frac{W_2^2}{2} \tag{1.11}$$

and thus the total voltage across the junction is

$$\psi_0 + V_R = V_1 + V_2 = \frac{q}{2\epsilon}(N_A W_1^2 + N_D W_2^2) \tag{1.12}$$

Substitution of (1.2) in (1.12) gives

$$\psi_0 + V_R = \frac{qW_1^2 N_A}{2\epsilon}\left(1 + \frac{N_A}{N_D}\right) \tag{1.13}$$

From (1.13), the penetration of the depletion layer into the p-type region is

$$W_1 = \left[\frac{2\epsilon(\psi_0 + V_R)}{qN_A\left(1 + \dfrac{N_A}{N_D}\right)}\right]^{1/2} \tag{1.14}$$

Similarly,

$$W_2 = \left[\frac{2\epsilon(\psi_0 + V_R)}{qN_D\left(1 + \dfrac{N_D}{N_A}\right)}\right]^{1/2} \tag{1.15}$$

Equations 1.14 and 1.15 show that the depletion regions extend into the p-type and n-type regions in *inverse* relation to the impurity concentrations and in proportion to $\sqrt{\psi_0 + V_R}$. If either N_D or N_A is much larger than the other, the depletion region exists almost entirely in the *lightly doped* region.

■ **EXAMPLE**

An abrupt pn junction in silicon has doping densities $N_A = 10^{15}$ atoms/cm^3 and $N_D = 10^{16}$ atoms/cm^3. Calculate the junction built-in potential, the depletion-layer depths, and the maximum field with 10 V reverse bias.

From (1.1)

$$\psi_0 = 26 \ln \frac{10^{15} \times 10^{16}}{2.25 \times 10^{20}} \text{ mV} = 638 \text{ mV} \quad \text{at} \quad 300°\text{K}$$

From (1.14) the depletion-layer depth in the p-type region is

$$\begin{aligned} W_1 &= \left(\frac{2 \times 1.04 \times 10^{-12} \times 10.64}{1.6 \times 10^{-19} \times 10^{15} \times 1.1}\right)^{1/2} = 3.5 \times 10^{-4}\text{cm} \\ &= 3.5\ \mu\text{m} \ \ (\text{where } 1\ \mu\text{m} = 1 \text{ micrometer} = 10^{-6} \text{ m}) \end{aligned}$$

The depletion-layer depth in the more heavily doped n-type region is

$$W_2 = \left(\frac{2 \times 1.04 \times 10^{-12} \times 10.64}{1.6 \times 10^{-19} \times 10^{16} \times 11}\right)^{1/2} = 0.35 \times 10^{-4} \text{ cm} = 0.35\ \mu\text{m}$$

Finally, from (1.7) the maximum field that occurs for $x = 0$ is

$$\begin{aligned} \mathscr{E}_{\text{max}} &= -\frac{qN_A}{\epsilon} W_1 = -1.6 \times 10^{-19} \times \frac{10^{15} \times 3.5 \times 10^{-4}}{1.04 \times 10^{-12}} \\ &= -5.4 \times 10^4 \text{ V/cm} \end{aligned}$$

■ Note the large magnitude of this electric field.