

Figure 4.27: AC-coupled comparator, a circuit that is very sensitive to charge errors.

4.2.5 Charge Conservation

There are several mechanisms that affect charge conservation. The charge conserving nature of semiconductor models is the most important factor that affects charge conservation because of the large amount of charge that is created or annihilated on every time step by capacitance-based models. The Meyer capacitance model [meyer71] used in MOS1, MOS2, and MOS3 models is capacitance-based and so is not charge conserving. The Ward-Dutton model [ward78], available in MOS2, the Yang-Chatterjee model [yang82], and the BSIM model, both available on all MOSFETs installed in Spectre, are all charge-based and so conserve charge.

Even when exclusively using charge-conserving models, there can still be some problems with charge conservation. The error due to the convergence criteria was discussed in the section on DC analysis. This results in charge not being completely conserved, which is an important point. Even simulators with charge-conserving models do not conserve charge completely, though they do a much better job of it than simulators that use non-charge-conserving models.

While charge conservation errors are small, they occur on each time step and in sensitive circuits, such as charge-storage circuits, the small charge errors can accumulate until they become a big problem. Consider the AC-coupled comparator shown in Figure 4.27. This is an example of a circuit that is extremely sensitive to error

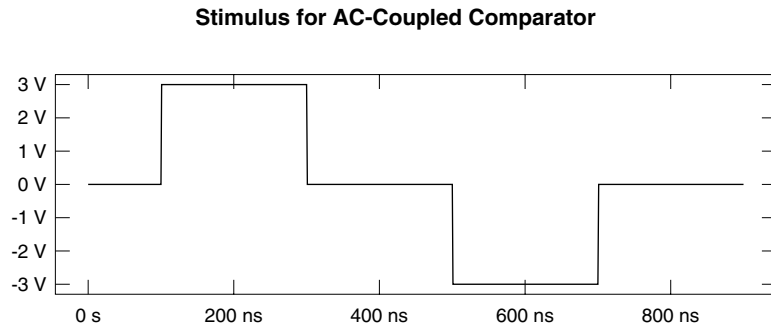


Figure 4.28: Stimulus waveform for the AC-coupled comparator.

(see Section 4.2.4 on page 160 to recall how certain types of circuits cause errors to accumulate). The switches in this circuit are initially set such that the offset in the amplifiers is stored on the interstage capacitors. Ideally, when switched back to the normal mode of operation, the offset stored on the capacitor cancels the offset on the amplifiers. Unfortunately, if the capacitor model used in the FETs acting as switches do not conserve charge, the charge error is stored on the interstage capacitors and amplified. This is shown in Figure 4.29, which is the output of the comparator when driven with the input shown in Figure 4.28. Since the stimulus returns to zero at the end of the simulation interval, the output should at least return to being close to zero. The charge error that results from using non-charge-conserving models prevents this from occurring. When the circuit is resimulated using charge conserving models, the computed solution (as shown in Figure 4.30 on the next page) is considerably more accurate, though it does exhibit some charge error. If `reltol` is tightened and the circuit resimulated, as shown in Figure 4.31 on page 170, all hint of charge error is gone.

Why Capacitance-based Models do not Conserve Charge In SPICE, the Meyer capacitance model is used to model the dynamic nature of the MOS devices. The Meyer model consists of equations that give capacitance as a function of terminal voltages. The capac-

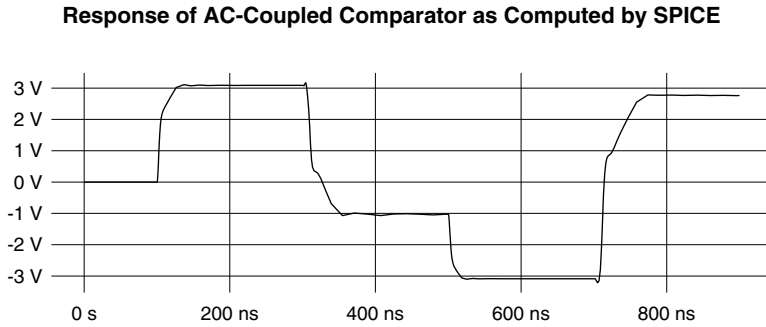


Figure 4.29: Response of the AC-coupled comparator computed by SPICE with non-charge conserving models. Notice that solution does not return to zero as it should.

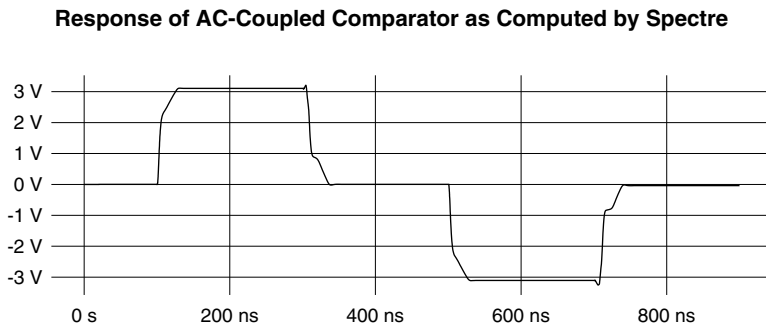


Figure 4.30: Response of the AC-coupled comparator computed by Spectre with charge conserving models. Notice that solution returns almost to zero.

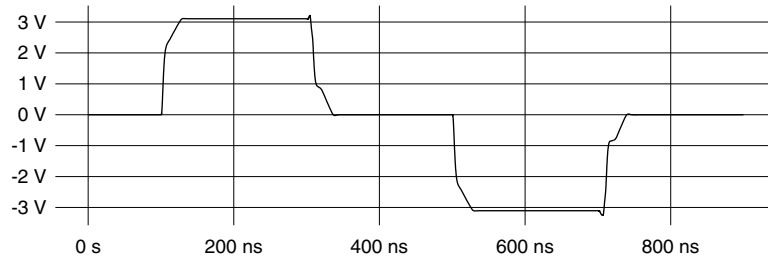
Response of AC-Coupled Comparator when Computed with Tight Tolerances


Figure 4.31: Response of the AC-coupled comparator computed by Spectre with `reltol` tightened. Notice that solution returns to zero as it should.

itors are nonlinear. Capacitance is an incremental or small-signal quantity that is defined as $C = dq/dv$. Defining a capacitor model by giving capacitance as a function of voltage results in the model not conserving charge. The reason capacitance-based models do not conserve charge is that capacitance is an incremental quantity that only accurately predicts the change in charge versus voltage for infinitesimally small changes in voltage. The other method of defining a capacitor model, that of giving charge as a function of voltage, does conserve charge.

Consider the following experiment, connect a linear $1\mu\text{F}$ capacitor across a voltage source. Assume that the voltage across the source changes from 0 to 1mV in one time step, and then back to zero on the next as shown in Figure 4.32. Let both time steps be 1ns . Then current is computed on the first step as $i = Cdv/dt = 1\text{A}$, and the charge is $Q = Cv = 1\text{nCoul}$. On the second step, the charge flow is equal and opposite, with a net zero charge being supplied by the source after the second step. Thus, capacitance based models do not result in charge conservation problems when the capacitor is linear and the capacitance is exact.

If the capacitor is nonlinear and described with an equation that

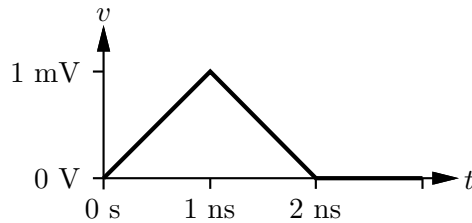


Figure 4.32: Stimulus waveform to the charge conservation example.

gives capacitance as a function of voltage, neither the current nor the charge can be computed exactly. They can be approximated using backward Euler (4.9) as follows,

$$i(t_1) = C(v(t_0))[v(t_1) - v(t_0)]/[t_1 - t_0] \quad (4.30)$$

$$\Delta q(t_1) = C(v(t_0))[v(t_1) - v(t_0)] \quad (4.31)$$

Thus if $C(0) = 1\mu\text{F}$ and $C(0.001) = 1.1\mu\text{F}$, then

$$i(t_1) = 1\text{A} \quad (4.32)$$

$$\Delta q(t_1) = 1\text{nCoul} \quad (4.33)$$

$$i(t_2) = -1.1\text{A} \quad (4.34)$$

$$\Delta q(t_2) = -1.1\text{nCoul} \quad (4.35)$$

Thus, the net charge is -0.1 nCoul . This is illustrated in Figure 4.33 on the next page.

If a charge-based model is used, the problem goes away. Assume the capacitor is modeled with the equation $q(v) = Cv + Dv^2$, where $C = 1\mu\text{F}$ and $D = 50\mu\text{F}/v$. This equation was chosen because it satisfies $C(0) = 1\mu\text{F}$ and $C(0.001) = 1.1\mu\text{F}$. Now the current still has to be approximated, but the approximation is such that charge is always conserved.

$$i(t_1) = [q(v(t_1)) - q(v(t_0))]/[t_1 - t_0] \quad (4.36)$$

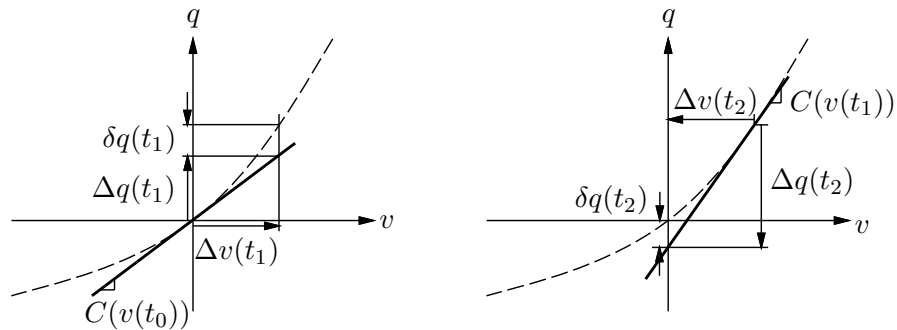


Figure 4.33: Example that shows that charge is not conserved (in other words, $\Delta q(t_1) \neq -\Delta q(t_2)$ even though $\Delta v(t_1) = -\Delta v(t_2)$) when using capacitance-based models because the change in charge on a time step is computed from a linear approximation of the charge function rather than the using the charge function itself. $\delta q(t_n)$ represents the charge error made on step n .

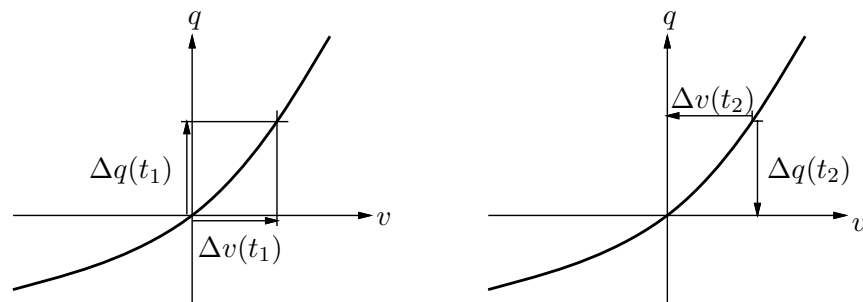


Figure 4.34: Example that shows that charge is conserved when using charge-based models (because $\Delta q(t_1) = -\Delta q(t_2)$ when $\Delta v(t_1) = -\Delta v(t_2)$).

$$i(t_1) = 1.05\text{A} \quad (4.37)$$

$$\Delta q(t_1) = 1.05\text{nCoul} \quad (4.38)$$

$$i(t_2) = -1.05\text{A} \quad (4.39)$$

$$\Delta q(t_2) = -1.05\text{nCoul} \quad (4.40)$$

The net charge is 0. This is illustrated in Figure 4.34 on the facing page.

The crucial point here is that since charge is computed explicitly, if a charge-based capacitor model is evaluated for the charge at a particular voltage, and then the voltage on the capacitor goes through some large excursion but returns to its original value, then the charge must also return to its original value because it is computed with an algebraic expression that has not changed using the same input. With a capacitance-based model, the small errors made on each step of the voltage trajectory accumulate and the final charge does not equal the starting charge.

This example also partially illustrates why the MOSFET capacitor models in SPICE have not been fixed. It is first necessary to derive a charge equation that fits the model. In other words, it is necessary to derive a set of charge relations for each terminal such that the derivative of these charges match the Meyer capacitances. Unfortunately, that is not possible. The Meyer capacitances are incomplete and inconsistent and so there exists no charge functions that when differentiated give the Meyer capacitances.

If you are forced to use the Meyer model, the charge-conservation problem can be reduced by tightening `reltol`, but it cannot be eliminated. Tightening `reltol` reduces charge error because it results in the simulator taking smaller time steps, which implies the change in voltage across the capacitor over a time step is smaller and so the linearized capacitance model is more accurate. However, Meyer's incomplete set of capacitances does result in charge being created or destroyed regardless of how small a time step is used.

The Statz GaAsFET model, unlike the Meyer model, does conserve total device charge because it has an explicit relationship between total charge and the terminal voltages. However, the model does not have explicit relationships for the drain and source charge, so charge

does mysteriously appear or disappear on the drain and source. Total charge is conserved because the explicit total-charge relationship forces the exact amount of charge that disappears from the drain to appear on the source and visa versa.

Requirements for Capacitor Models to Conserve Charge³

For a capacitor model to be charge conserving, it must be such that if the voltage is changed and then returned to its original value, the final charge must equal the initial charge, regardless of the path taken or the starting point. This is true for models described with single-valued charge functions because $q(v_i) = q(v_f)$ if $v_i = v_f$. As shown previously, capacitance-based models do not conserve charge if the capacitor is nonlinear and the path is discretized with a finite number of steps because the capacitance is a linear approximation to the charge function and if the steps are not infinitesimal, a finite error accumulates on each step.

An interesting question remains. If only a capacitance-based model is available, can the charge error be reduced to negligible levels by taking small time steps. For a capacitance-based model to be charge conserving when the time step asymptotically approaches zero the capacitances must include all of the derivatives of a charge function, and must be exact. This is shown by integrating all the charge that flows into a capacitor while traversing a closed path of voltages. Consider the charge that flows from gate-to-source on a MOSFET. This charge is a function of V_{gs} , V_{gd} , and V_{gb} and so is denoted $Q_{gs}(V_{gs}, V_{gd}, V_{gb})$. In the notation of vector calculus, the total charge accumulated around the path is written

$$\oint_C \frac{\partial Q_{gs}}{\partial V} dV = \oint_C \nabla Q_{gs}(V) \cdot dV = Q_{\text{err}} \quad (4.41)$$

where $V = [V_{gs} \ V_{gd} \ V_{gb}]^T$ is the voltage vector, $\nabla Q_{gs}(V)$ is the capacitance written as a gradient, C is any closed path, and Q_{err} is the total charge accumulated along the path. Expanding the gradient

³This section assumes the reader has a background in vector calculus. You may skip this section without loss of continuity.

gives

$$\oint_C \left(\frac{\partial Q_{gs}}{\partial V_{gs}} dV_{gs} + \frac{\partial Q_{gs}}{\partial V_{gd}} dV_{gd} + \frac{\partial Q_{gs}}{\partial V_{gb}} dV_{gb} \right) = Q_{\text{err}}. \quad (4.42)$$

Stoke's theorem states that

$$\oint_C \nabla Q_{gs}(V) \cdot dV = \int_S \nabla \times \nabla Q_{gs} \cdot dS \quad (4.43)$$

where S is the surface enclosed by C . Using Stoke's theorem on (4.41) converts the path integral in terms of the first derivatives of charge to a surface integral in terms of the second derivatives of charge.

$$\int_S \nabla \times \nabla Q_{gs} \cdot dS = Q_{\text{err}} \quad (4.44)$$

Expanding the cross and dot products gives

$$\begin{aligned} & \int_S \left(\frac{\partial^2 Q_{gs}}{\partial V_{gd} \partial V_{gb}}(V_{gs}, V_{gd}, V_{gb}) - \frac{\partial^2 Q_{gs}}{\partial V_{gb} \partial V_{gd}}(V_{gs}, V_{gd}, V_{gb}) \right) dS_{gs} + \\ & \left(\frac{\partial^2 Q_{gs}}{\partial V_{gb} \partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}) - \frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gb}}(V_{gs}, V_{gd}, V_{gb}) \right) dS_{gd} + \\ & \left(\frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gd}}(V_{gs}, V_{gd}, V_{gb}) - \frac{\partial^2 Q_{gs}}{\partial V_{gd} \partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}) \right) dS_{gb} = Q_{\text{err}} \end{aligned} \quad (4.45)$$

If Q_{gs} is continuously differentiable, it can be shown that the second derivative must be symmetric⁴. This condition requires that

$$\frac{\partial^2 Q_{gs}}{\partial V_{gd} \partial V_{gb}}(V_{gs}, V_{gd}, V_{gb}) = \frac{\partial^2 Q_{gs}}{\partial V_{gb} \partial V_{gd}}(V_{gs}, V_{gd}, V_{gb}) \quad (4.46)$$

$$\frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gd}}(V_{gs}, V_{gd}, V_{gb}) = \frac{\partial^2 Q_{gs}}{\partial V_{gd} \partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}) \quad (4.47)$$

$$\frac{\partial^2 Q_{gs}}{\partial V_{gb} \partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}) = \frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gb}}(V_{gs}, V_{gd}, V_{gb}). \quad (4.48)$$

⁴It is a standard theorem in vector calculus that if a function is continuously differentiable at a point, then its second derivative or Hessian must be symmetric at that point.

Substituting (4.46–4.48) into (4.45) gives

$$0 = Q_{\text{err}}. \quad (4.49)$$

Thus, the charge accumulated along a closed-path of voltages for a capacitance-based model is zero if the steps taken to traverse the path are infinitesimally small, if the capacitances actually stem from some charge function, if they include all of the derivatives of the charge function, and if they are exact.

A complete set of derivatives for $Q_{gs}(V_{gs}, V_{gd}, V_{gb})$ include

$$\frac{\partial Q_{gs}}{\partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}) = C_{gs}(V_{gs}, V_{gd}, V_{gb}) \quad (4.50)$$

$$\frac{\partial Q_{gs}}{\partial V_{gd}}(V_{gs}, V_{gd}, V_{gb}) \quad (4.51)$$

$$\frac{\partial Q_{gs}}{\partial V_{gb}}(V_{gs}, V_{gd}, V_{gb}). \quad (4.52)$$

Of this set, the Meyer capacitance model only provides C_{gs} , the other derivatives are neglected. This is insufficient to conserve charge because (4.47–4.48) are not satisfied. In fact, since $\frac{\partial Q_{gs}}{\partial V_{gd}}$ is zero in the Meyer model, then $\frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gd}}$ is zero for all V_{gs} , V_{gd} , and V_{gb} . C_{gs} is a function of V_{gd} , so $\frac{\partial Q_{gs}}{\partial V_{gd}} C_{gs} = \frac{\partial^2 Q_{gs}}{\partial V_{gd} \partial V_{gs}}$ is nonzero. Thus,

$$\frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gd}}(V_{gs}, V_{gd}, V_{gb}) \neq \frac{\partial^2 Q_{gs}}{\partial V_{gd} \partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}). \quad (4.53)$$

Similarly,

$$\frac{\partial^2 Q_{gs}}{\partial V_{gs} \partial V_{gb}}(V_{gs}, V_{gd}, V_{gb}) \neq \frac{\partial^2 Q_{gs}}{\partial V_{gb} \partial V_{gs}}(V_{gs}, V_{gd}, V_{gb}). \quad (4.54)$$

Therefore, there is no charge function that has a derivative that equals the Meyer capacitance. Also, since important terms in (4.45) are neglected, $Q_{\text{err}} \neq 0$ and the model does not conserve charge, regardless of how little the voltage changes on each time step.