A Refined Nonlinear Averaged Model for Constant Frequency Current Mode Controlled PWM Converters

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***Abstract*—Existing nonlinear averaged models for current programmed converters are based on a steady-state formulation of the current mode control law. Although reasonably accurate at the lower frequencies ($f \ll f_{\text{switching}}$), these models tend to break down well before sampled-data effects nullify the assumptions underlying state-space averaging. Moreover, the SPICE implementation of the large-signal models is often plagued by a multitude of simulation problems. In this paper, a refined duo-model model for current programmed PWM converters is presented which exhibits improved high frequency accuracy over previous formulations and whose SPICE implementation remains, thus far, remarkably free of simulation problems. The large-signal transient response predicted using the refined average model is shown to be virtually indistinguishable, in an average sense, from that predicted using a pulse-by-pulse simulation.**

I. Introduction

Since its inception, current mode control has gained increasing acceptance among designers of dc-to-dc PWM converters. Its popularity, no doubt enhanced by the number of advantages it offers over the more conventional duty-ratio control scheme [1], [2]. Two averaged models have been presented [3], [4] that claim to characterize the large and small signal behavior of current-programmed PWM converters under both continuous conduction mode (CCM) and discontinuous conduction mode (DCM) operation. In [3], the authors developed a duo-mode model for a current programmed buck power stage based on a mode invariant (CCM and DCM) formulation of the current mode control law. However, there, and to the authors' regret, the presentation was somewhat obscured by the complexity of an implementation plagued with simulation (convergence) problems. More recently, Griffin [4] presented duo-mode models for current programmed power stages that use diode-OR logic to switch between the laws governing CCM and DCM operation. However, there too, the author hints at simulation problems even though all comparative testing was conducted open loop, with no input filter and in the frequency domain. It appears that attempts at duo-mode modeling, more often than not, tax the limits of the simulator, not to mention the patience of the modeler. Although reasonably accurate at the lower frequencies ($f \ll f_{\text{switching}}$), the previous models tend to break down well before sampled-data effects [5] nullify the assumptions underlying state-space averaging. Moreover, the SPICE [6] implementation of these models is often plagued by a multitude of simulation problems.

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In this paper, a refined, duo-mode model for current programmed buck converters is presented which has proven, thus far, to be remarkably free of simulation problems. The refined model uses a form of the current mode control law which is truly invariant with respect to operating conditions. That is, it is valid for both transient and steady-state operating conditions regardless of the converter operating mode (CCM or DCM). Hence, it removes the inconsistency present in previous models based on the steady-state formulation of the current mode control law and allows for a much simpler SPICE implementations of duo-mode operation. Equally significant is the utilization of a new and more general expression for the substitute variable used to replace the state variable "lost" in modeling DCM operation. The refined model is shown to exhibit improved high frequency accuracy in both time and frequency domains. The model has been implemented in SPICE 2G6 and runs with default analysis options.

The development begins in the next section with a review of large signal CCM modeling in SPICE.

II. Modeling Continuous Conduction Operation

Large signal CCM modeling of switched dc/dc constant frequency, current programmed converters in SPICE usually follows the approach presented by Bello [7], which in turn is based on the well known circuit and state space averaging methods developed by Wester, Middlebrook and Ćuk [8], [9]. In [7], Bello combined the non-linear versions of the canonical model for the basic converter power stages with the current mode control law to arrive at large signal models for current programmed converters operating in the continuous conduction mode. The structure of the resulting model is depicted in Fig. 1, where the transformer, inductor and load constitute the circuit realization of the state space averaged equations for the buck converter and the block labeled PWM enforces the

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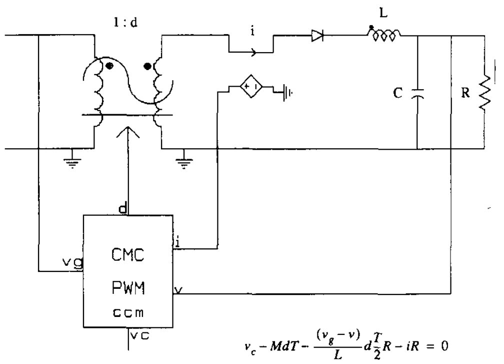


Fig. 1. Large signal model for current mode buck converter operating in the continuous conduction mode.

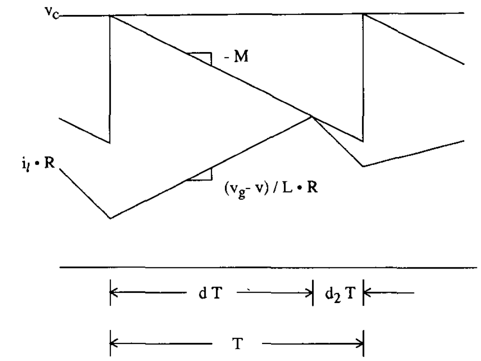


Fig. 2. Control voltage, stabilizing ramp and inductor current waveforms during transient conditions (CCM).

current mode control law relating the control voltage, v_c, and the duty ratio, d, to the average inductor current, i, according to

$$v_c - MdT - \frac{(v_g - v)}{2L} dTR = iR \tag{1}$$

where v_g, v, M, L and T denote the input voltage, output voltage, stabilizing ramp slope, output inductance and switching period, respectively. The constant R represents the gain associated with the inductor current sensing scheme. Recently, Verghese, *et al.* discovered an inconsistency in the derivation of the previous equation [10]. The derivation is based on the waveforms appearing at the input of the PWM during steady-state operation; i.e., $f(t) = f(t + T)$. Nevertheless (1) is presumed to be valid during transient conditions as well. Fig. 2 depicts the control voltage, stabilizing ramp and inductor current waveforms appearing at the input of the PWM during transient conditions. An inspection of Fig. 2 reveals that a more general expression for the current mode control law is given by

$$v_c - MdT - \frac{d(v_g - v)\, dTR}{2L} - \frac{d'(v)\, d'TR}{2L} = iR \tag{2}$$

where $d' = 1 - d$. It should be evident that in the steady-state (2) reduces to (1). In [10], Verghese used small signal ac analysis of a typical boost converter to demonstrate that the previous inconsistency results in non-trivial differences in the location of the open loop poles. Although both (1) and (2) result in open loop transfer functions that predict the same low frequency (dominant) pole, the transfer function resulting from the more general and accurate expression for the current mode control law places the second pole at a higher frequency than the high frequency pole resulting from the application of the steady-state current mode control law. Consequently, it appears that the use of the steady-state current mode control law could result in an incorrect characterization of the converter high frequency dynamics. Fortunately, the error usually results in a conservative assessment of the converter transient performance. Nevertheless, in applications that demand a high level of accuracy, the error becomes significant enough that it cannot be neglected.

The next section examines DCM modeling. There, it will be observed that the current mode control law derived from the steady state inductor current waveforms is also valid under transient conditions if the current, i, in (1) is understood to be the average over the inductor conduction interval.

III. Modeling Discontinuous Conduction Operation

Discontinuous conduction operation of current programmed PWM converters is similar to that of duty-ratio programmed converters operating in the discontinuous conduction regime, as evidence by the single (dominant) pole characterizing the dynamics of both converters. Hence, it should not come as a surprise that large signal DCM modeling of current programmed PWM converters in SPICE follows the modeling of DCM operation in duty-ratio programmed converters [11].

It is instructive to review briefly some important facts concerning DCM operation. Consider Fig. 3, depicting the inductor current in a PWM converter operating with discontinuous inductor current during transient conditions. An inspection of Fig. 3 reveals fundamental differences between continuous and discontinuous mode operation. Most significant is the observation that the boundary conditions for the inductor current are fixed at zero. That is,

$$i_l[0] = i_l[(d + d_2)T] = i_l[T] = 0 \tag{3}$$

Furthermore, averaging of the state equations for the DCM converter over a switching interval results in an expression of the form

$$\begin{bmatrix} 0 \\ \dot{v} \end{bmatrix} = f(i, v, v_g, d, d_2) \tag{4}$$

where f is the nonlinear, continuous function resulting from the averaging of the state equations corresponding to the three switched networks associated with DCM op-

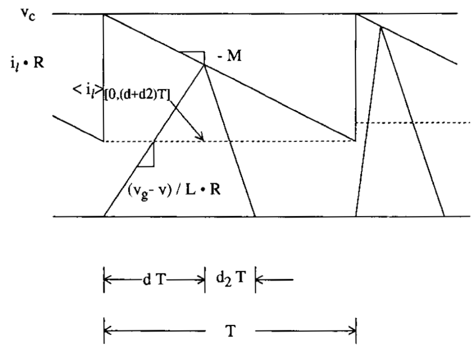


Fig. 3. Control voltage, stabilizing ramp and inductor current waveforms during transient conditions (DCM).

eration. Equation (4) appears to be consistent with the constraint

$$\frac{di}{dt} = 0 \tag{5}$$

Some researchers [12] use this observation to disqualify the inductor current as a legitimate state variable and consequently, exclude the inductor from the resulting model. Others [13], argue that there is no theoretical justification for the disappearance of the inductor current state and choose to retain the inductor in the resulting circuit model. Still others [14], have elected to discard the constraint (5) from the derivation of the state-space averaged model. Here, the authors choose to follow the reduced-order approach described in [12]. In any case, (5) should not be construed to mean that the averaged inductor current is constant. In fact an inspection of Fig. 3 reveals that quite the opposite is true; i.e.,

$$\frac{d}{dt}\frac{1}{T}\int_{t}^{(t+T)} i_l(\tau)\, d\tau \neq 0 \tag{6}$$

The zero on the L.H.S. of (4) is strictly the result of the fixed boundary conditions; i.e.,

$$\frac{1}{T}\int_{kT}^{(k+1)T} \frac{d}{dt} i_l(t)\, dt = \frac{i_l[(k+1)T] - i_l[kT]}{T} = 0 \tag{7}$$

and does not imply a constant averaged inductor current. Equation (5) is only meant to indicate that the averaged inductor current has lost its dynamic character; i.e., it has ceased to be a true state variable. This is in sharp contrast with CCM operation where (5) implies steady-state conditions. In order to solve (4), a substitute variable is introduced to replace the lost state. The substitute variable most commonly used is the averaged inductor current defined by

$$i = \frac{(v_g - v)}{L} d \frac{T}{2} = \frac{v}{L} d_2 \frac{T}{2}. \tag{8}$$

Observe that (8) is an algebraic equation relating the substitute variable to the remaining state variable and the inputs; hence, it is consistent with (5). Henceforth, (8) will be referred to as the auxiliary equation for the discontinuous mode.

The nonlinear, circuit averaged model for a buck power stage under DCM operation is depicted in Fig. 4. The fictitious step-down and step-up transformers, the inductor and the load (not shown) constitute the circuit realization of the state space averaged equations for a buck converter operating with discontinuous inductor current [12]. The inductor is shown shorted as implied by (5). The turns ratio of the step-up transformer must be controlled so that it enforces the equality

$$\langle i_l \rangle_{[0,T]} = (d + d_2)\langle i_l \rangle_{[0,(d+d2)T]} \tag{9}$$

where

$$\langle i_l \rangle_{[0,\tau]} = \frac{1}{\tau}\int_0^{\tau} i_l(t)\, dt \tag{10}$$

Equation (9) states that the state space averaged inductor current is equal to the inductor current averaged over the inductor conduction interval times the quantity $d + d_2$. The quantity d_2, associated with the inductor discharge time, may be obtained by solving (8) for d_2. The block labeled PWM enforces the current mode control law relating the control voltage, v_c, stabilizing ramp, M, and duty ratio, d, to the averaged inductor current, i according to (1). Observe that both (1) and (8) are valid under steady-state and transient conditions (within the discontinuous conduction regime) if the current, i, in (1) is understood to be the average over the inductor conduction interval.

In the next section, a mode invariant form of the corrected CCM current mode control law and a revised auxiliary equation are introduced which together with the DCM power stage model of Fig. 4 lead to the formulation of a refined, nonlinear duo-mode model with improved high-frequency accuracy.

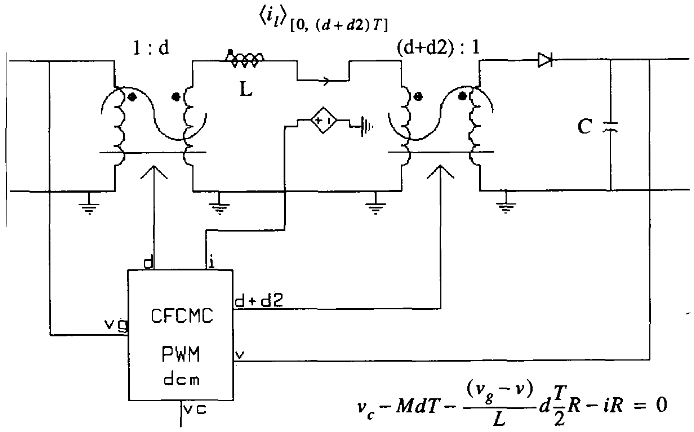
IV. Bridging the Gap: Duo-Mode Modeling

The duo-mode model presented herein relies on the existence of an expression for the current mode control law which is valid regardless of the converter conduction mode *and* operating condition (transient or steady-state). In Section II, a general expression for the current mode control law was presented in the form of (2) repeated below for convenience:

$$v_c - MdT - \frac{d(v_g - v)\, dTR}{2L} - \frac{d'(v)\, d'TR}{2L} = iR. \tag{11}$$

The previous equation was derived based on the inductor current, control voltage and stabilizing ramp waveforms during transient conditions. An examination of those same waveforms (Figs. 2 and 3) reveals that a mode invariant form of (11) may be expressed as

$$[v_c - MdT](d + d_2) - \frac{d(v_g - v)\, dTR}{2L} - \frac{d_2(v)\, d_2TR}{2L} = iR. \tag{12}$$



$$v_c - MdT - \frac{(v_g - v)}{L} d \frac{T}{2} R - iR = 0$$

$$i = \frac{(v_g - v)}{L} d \frac{T}{2}$$

Fig. 4. Large signal model for current mode buck converter operating in discontinuous conduction mode.

Equation (12) is valid for both transient *and* steady-state operating conditions regardless of the converter operating mode (CCM or DCM). Hence, it is reasonable to expect that (12) will lead to a more accurate characterization of the dynamics of current mode controlled PWM converters. In subsequent developments, (12) will be referred to as the refined current mode control law. The SPICE implementation of (12) is described in the Appendix. Here, it suffices to say that given values for the variables v_c, v_g, v, i and d_2, and for the constants M, L, R and T, (12) may be solved for the duty ratio, d, subject to the constraint $0 \leq d \leq 1$.

One final issue must be addressed before the new model can be presented. In Section III, the average inductor current $\langle i_l \rangle_{[0,(d+d2)T]}$ was introduced as a replacement for the lost state variable. Here, it is argued that since it is desired that the trajectory of the averaged inductor current approximates the moving average along the trajectory of the instantaneous inductor current, a better substitute variable is given by

$$\frac{1}{T}\int_t^{(t+T)} i_l(\tau)\, d\tau \approx \frac{1}{T}\int_{kT}^{(k+1)T} i_l(t)\, dt$$

$$= d^2 T \frac{(v_g - v)}{2L} + d_2^2 T \frac{v}{2L} = i \quad (13)$$

The equalities in (13) are valid for both steady state and transient conditions, *but only* under DCM operation. Evidently, the expression needs to be modified to accommodate CCM operation. A revised form of (13) amenable to the implementation of a duo-mode model is given by

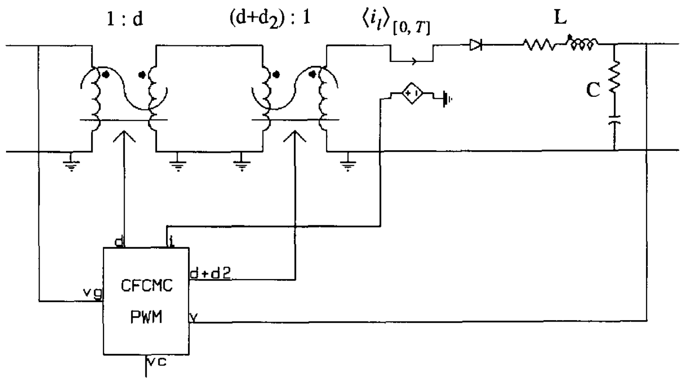
$$i = d^2 T \frac{(v_g - v)}{2L} + d_2^2 T \frac{(v)}{2L}; \qquad DCM$$

$$\frac{di}{dt} = f(i, v, v_g, d); \qquad d_2 = 1 - d; \qquad CCM. \quad (14)$$

Equation (14) reaffirms the fact presented in Section III regarding the static nature of the average inductor current during DCM operation. Moreover, it states that during CCM operation, the average inductor current is free to vary according to the control strategy (current or duty-ratio programming). The SPICE implementation of (14) is described in the Appendix. Henceforth, (14) will be referred to as the refined auxiliary equation.

The structure of the new large signal model for current mode controlled PWM converters is shown in Fig. 5 for the case of a step-down (buck) dc/dc converter. The model is an extension of the non-linear circuit averaged model for the DCM presented in Section III. The block labeled PWM enforces the refined current mode control law (12) and the refined auxiliary equation (14). During converter operation (CCM or DCM), the turns ratio of the fictitious step-down transformer is varied to maintain the averaged inductor current, i, at the value given by the refined current mode control law. The turns ratio of the step-up transformer, fixed at unity during CCM operation, varies during DCM operation to maintain the average inductor current at the value given by the refined auxiliary equation. The inductor follows the step-up transformer because that is the location that results in the correct power dissipation in the inductor when conduction losses are considered. However, unlike the development in [11], [12], the inductor is not shorted during DCM operation. The reason for this is quite simple. Since current is being forced through the inductor according to (14), the inductor has been effectively removed from the circuit. More rigorously stated, (14) is an algebraic equation (no dynamics) relating the inductor current to the only remaining state variable and the inputs; hence, the inductor current has ceased to be a true state variable. Interestingly enough, the same can be said of current programming yet the inductor is never shorted in large signal, uni-mode (CCM) models.

$$i = d^2 T \frac{(v_g - v)}{2L} + d_2^2 T \frac{(v)}{2L}; \quad \text{DCM}$$



$$[v_c - MdT]\,(d + d_2) - \frac{d\,(v_g - v)\,dTR}{2L} - \frac{d_2\,(v)\,d_2 TR}{2L} = iR$$

Fig. 5. Refined duo-mode model for a current mode buck converter.

Several significant differences exist between the refined and previous duo-mode models [3], [4]. The fundamental difference lies in the validity of the current mode control law on which all of them are based. The refined model uses a form of the current mode control law which is *truly invariant* with respect to operating conditions. That is, it is valid for both transient and steady-state operating conditions regardless of the converter operating mode (CCM or DCM). Hence, it removes the inconsistency present in all other models based on steady-state formulations of the current mode control law and allows for a much simpler implementation of duo-mode operation. For example, in [4], two current mode control laws, one for each operating regime, are diode-ORed together to permit duo-mode operation. Equally significant is the utilization of a new and more general expression for the substitute variable used to replace the state variable "lost" in modeling DCM operation. A consequence of the refinements to the current mode control law and to the auxiliary equation will be observed in the next section where the model is shown to exhibit improved accuracy in both time and frequency domains. Another important difference lies in the handling of the inductor during DCM. In [3], the authors went to great lengths to insure that the inductor was removed from the circuit during DCM operation. This action, although perhaps theoretically justified, turns out to be completely unnecessary and is to blame for a multitude of simulation problems. The latter can be appreciated by those SPICE modeling practitioners who have spent numerous hours trying to overcome convergence problems. The remaining differences are all in the area of the SPICE implementation and result in circuit complexity (size) and simulation speed (time) advantages over previous formulations. The model has been implemented in SPICE 2G6 and runs with default analysis options. Nodesetting of the converter output node is sufficient (but not necessary) to ensure convergence to the desired dc solution. The SPICE implementation of the refined model is discussed in detail in the Appendix.

V. Model Validation

In this section, the accuracy of the refined averaged model is established through direct comparison with a device level (exact) simulation of a typical buck converter. DC and frequency domain accuracies are emphasized through comparison with the CCM and DCM small signal canonical models [12], [15]. All subsequent analyses were conducted on a Sun 3/110 workstation running the Analog Workbench, a SPICE-based circuit simulation program from Valid Logic Systems, Inc.

Fig. 6 depicts the device level, SPICE simulation of the 500 W dc bus voltage regulator used to validate the refined averaged model. The regulator utilizes a current programmed, PWM, dc/dc converter to maintain the bus voltage at 28 VDC. The FET switch is turned-on when the 200-kHz oscillator sets the latch. Turn-off takes place when the comparator resets the latch; that is, when the sum of the stabilizing ramp and the switch current exceeds the output of the error amplifier. A behavioral level, SPICE simulation of the same 500 Watt dc bus regulator is depicted in Fig. 7. The simulation is based on the refined averaged model described in Section IV and uses the fictitious step-up and step-down transformers, together with the current-mode controller to implement the voltage regulation function. The transient response of the bus regulator to load perturbations that force the converter to operate in both light and heavy modes is depicted in Figs. 8 and 9. Shown in Figs. 8 and 9 are the predicted inductor current and capacitor (output) voltage, respectively, during and following load transients. Initially, the converter is delivering rated current (17.5 A) into a resistive load.

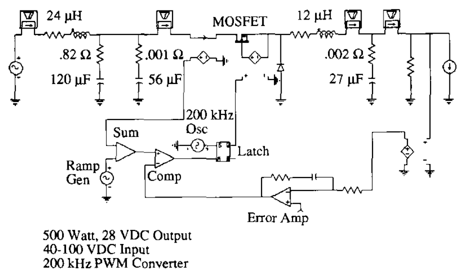


Fig. 6. Device level SPICE simulation of 500 W dc bus regulator using exact (discrete) model.

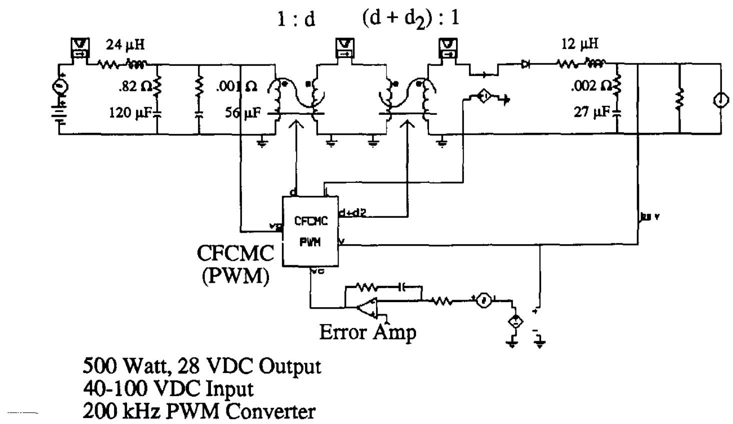


Fig. 7. Behavioral level SPICE simulation of 500 W dc bus regulator using refined averaged model.

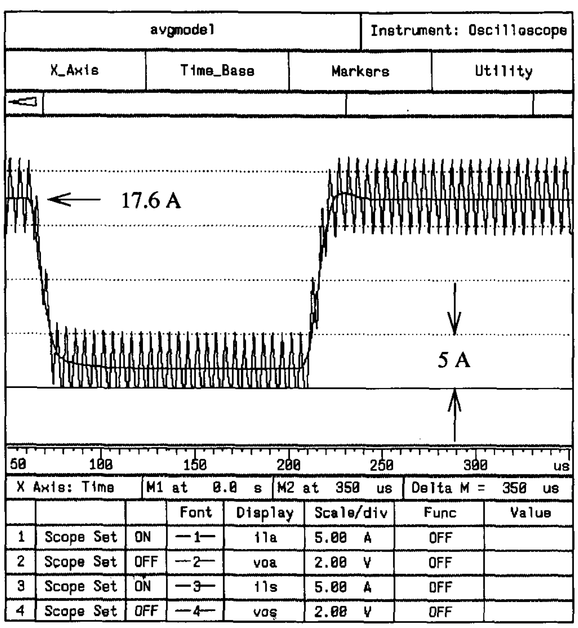


Fig. 8. Inductor current during and following step changes in load.

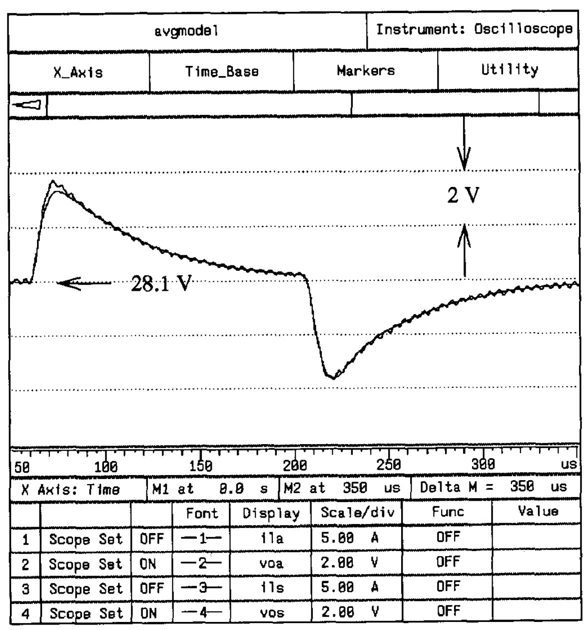


Fig. 9. Capacitor (output) voltage during and following step changes in load.

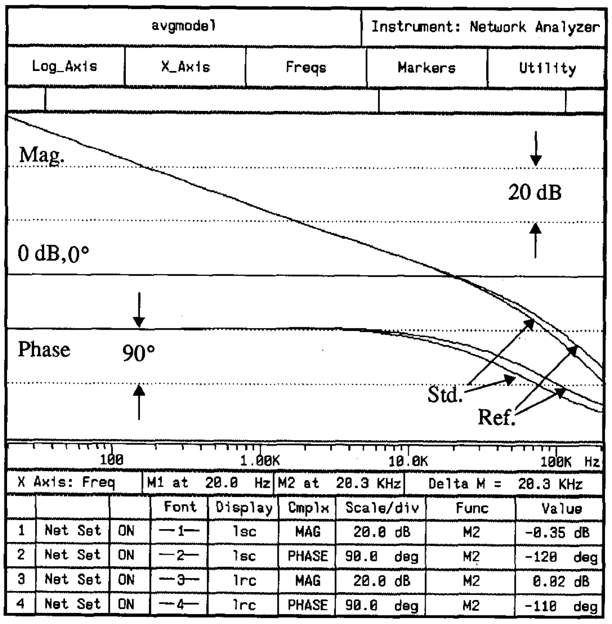


Fig. 10. Bode plots of predicted loop gain for 500 W dc bus regulator during CCM operation.

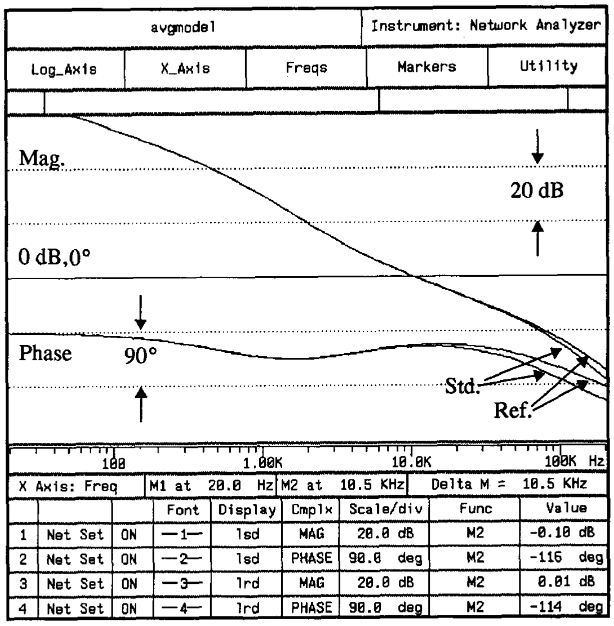


Fig. 11. Bode plots of predicted loop gain for 500 W dc bus regulator during DCM operation.

At time $t = 60$ μs, load shedding (15.5 A) causes the output voltage to increase momentarily; however, the regulator acts to bring the output voltage back to its nominal value. At time $t = 205$ μs, the load is reconnected and the output voltage now drops momentarily but is quickly returned to its nominal value by the action of the regulator. The response predicted by the refined model is shown to be in very close agreement with that predicted by the discrete model.

A frequency domain comparison between the refined and the standard canonical small-signal models is depicted in Fig. 10, which shows Bode plots of the open loop gain for the 500 W dc bus regulator during CCM operation. Fig. 11 shows Bode plots of the same transfer function but with the dc bus regulator operating deep into the discontinuous conduction regime. As expected, the difference between the refined and the standard averaged models is most significant at high frequencies corresponding to the difference in the location of the high frequency pole (CCM) and to the improved accuracy in the approximation of the moving average of the instantaneous inductor current (DCM).

VI. Conclusion

Until now, nonlinear averaged models for current programmed converters have been based on a steady-state formulation of the current mode control law. Although reasonably accurate at the lower frequencies ($f << f_{\text{switching}}$), these models tend to breakdown well before sample data effects nullify the assumptions underlying state-space averaging. In this paper, a refined duo-mode model for current programmed PWM converters has been presented which exhibits improved high frequency accuracy over previous formulations. In particular, the large-signal transient response predicted using the refined average model was shown to be virtually indistinguishable, in an average sense, from that predicted using a discrete (exact) model. Although the derivation of the model assumes a buck converter, the modeling process itself can easily be extended to include the boost and buck-boost converter topologies.

Appendix

Vital to the SPICE implementation of the refined averaged model is the computation of the duty ratio, d, from the refined current mode control law presented in Section IV and repeated below for convenience:

$$[v_c - MdT](d + d_2) - \frac{d(v_g - v)\, dTR}{2L} - \frac{d_2(v)\, d_2 TR}{2L} - iR = 0. \quad \text{(A1)}$$

Although highly nonlinear, it is possible to solve (A1) in SPICE by synthesizing a circuit whose mathematical description matches the above expression. To eliminate the possibility of undesirable roots (legitimate but physically impossible solutions) it is imperative to impose the following constraint:

$$0 \leq d \leq 1. \quad \text{(A2)}$$

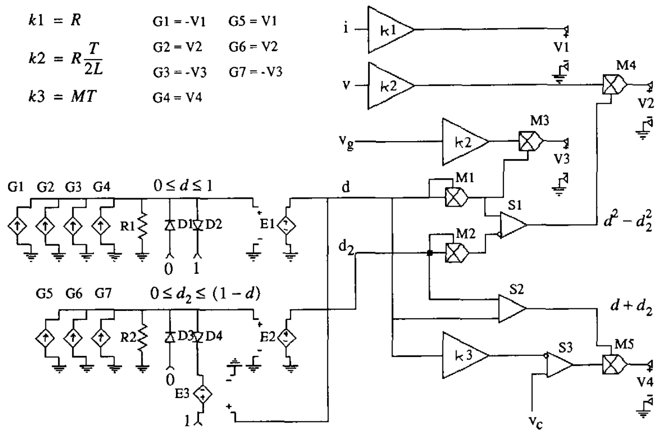


Fig. 12. SPICE implementation of duo-mode current programmed PWM.

Fig. 12 depicts the circuit used to solve (A1). The part of the circuit consisting of the constant gain elements $K_1 - K_3$, summers $S_1 - S_3$ and multipliers $M_1 - M_5$ is used to generate voltages corresponding to the five terms on the left hand side of (A1). These voltages, in turn, are fed-back to control the current generators $G_1 - G_4$. Since the sum of the currents flowing into a node must equal zero, the voltage appearing across R_1 represents the error in (A1) which is driven to zero by virtue of the high feedback gain. Diodes $D_1 - D_2$ and the controlled voltage generator E_1 restrict the solution space of (A1) by enforcing the constraint (A2). Observe that no extraneous circuit elements are present in this SPICE implementation; i.e., no fictitious circuit elements are introduced with the sole purpose of overcoming convergence problems.

In addition to the above, implementation of the refined averaged model in SPICE requires the computation of the quantity d_2 from the auxiliary equation given in Section IV and repeated below, albeit in a slightly different form:

$$\frac{d(v_g - v)\, dTR}{2L} + \frac{d_2(v)\, d_2 TR}{2L} - iR = 0. \qquad \text{(A3)}$$

The following constraint is imposed to eliminate the possibility of undesirable roots:

$$0 \le d_2 \le (1 - d). \qquad \text{(A4)}$$

The circuit used to solve (A3) is also shown in Fig. 12. The implementation exploits commonality with some of the terms in (A1). Thus, the voltages $V_1 - V_3$ generated in solving (A1) are also used in the solution of (A3). $V_1 - V_3$ correspond to the terms on the left hand side of (A3). These voltages are feedback to control the current generators $G_5 - G_7$. The voltage appearing across R_2 represent the error in (A3) which is driven to zero by virtue of the feedback. Diodes D_3, D_4 and the controlled sources E_2, E_3 enforce the constraint (A4).

The SPICE implementation of the variable turns ratio transformer used in the refined and standard averaged model is described in [11]. Copies of all the SPICE decks used in this work are available from the authors upon request.

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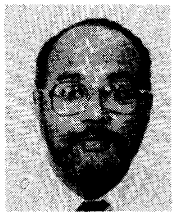

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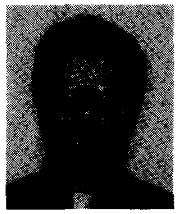

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