

for a square law device

analyzing a current loop at the bottom region of the text figure

$$R \cdot I + v_{gsM4} = v_{gsM1}$$

where

$$I = \frac{1}{2} \mu_n \cdot C_{ox} \cdot \frac{W \cdot 4}{L} \cdot (v_{gsM4} - v_t)^2$$

and

$$I = \frac{1}{2} \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (v_{gsM1} - v_t)^2$$

$$\sqrt{I} = \sqrt{\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W}{L} \right) \cdot (v_{gsM1} - v_t)^2}$$

solve for v_{gs}

$$v_{gsM1} = \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W}{L} \right) \right]}} + v_t$$

and

$$v_{gsM4} = \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W \cdot 4}{L} \right) \right]}} + v_t$$

returning to original loop equation and substituting

$$R \cdot I + \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W \cdot 4}{L} \right) \right]}} + v_t = \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W}{L} \right) \right]}} + v_t$$

neglecting body effect- v_t 's are the same for the same device

$$R \cdot I + \frac{1}{2} \cdot \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W}{L} \right) \right]}} = \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W}{L} \right) \right]}}$$

$$R \cdot I = \frac{1}{2} \cdot \sqrt{\frac{I}{\left[\frac{1}{2} \cdot \left(\mu_n \cdot C_{ox} \cdot \frac{W}{L} \right) \right]}}$$

simplifying above equation

$$R \cdot I = \frac{1}{2} \cdot \sqrt{\frac{2 \cdot I}{\mu n \cdot \text{Cox} \cdot \frac{W}{L}}}$$

remembering that

$$g_{m1} = \sqrt{2 \mu n \cdot \text{Cox} \cdot \frac{W}{L}} \cdot \sqrt{\frac{1}{2} \mu n \cdot \text{Cox} \cdot \frac{W}{L}} \cdot (v_{gsM1} - v_t)$$

$$\frac{g_{m1}}{\sqrt{\mu n \cdot \text{Cox} \cdot \frac{W}{L}}} = \sqrt{\frac{2 \mu n \cdot \text{Cox} \cdot \frac{W}{L}}{2}} \cdot (v_{gsM1} - v_t) = \sqrt{I \cdot 2}$$

solving for $\frac{1}{2\sqrt{\mu n \cdot \text{Cox} \cdot \frac{W}{L}}}$ gets

$$\frac{1}{\sqrt{\mu n \cdot \text{Cox} \cdot \frac{W}{L}}} = \frac{\sqrt{2 \cdot I}}{g_{m1}}$$

substituting above to below

$$R \cdot I = \frac{1}{2} \cdot \sqrt{\frac{2 \cdot I}{\mu n \cdot \text{Cox} \cdot \frac{W}{L}}} = \frac{I}{g_{m1}}$$

$$R = \frac{1}{g_{m1}}$$