Measuring AM, PM & FM Conversion with SpectreRF

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1 Derivation

Consider a sinusoid that is both amplitude and phase modulated simultaneously.

$$v_m(t) = A_c(1 + \alpha(t)) \cos(\omega_c t + \phi_c + \phi(t)) \tag{1}$$

where A_c, ϕ_c and ω_c are the amplitude, phase and angular frequency of the carrier, while $\alpha(t)$ and $\phi(t)$ are the amplitude and phase modulation. Assume that $\phi(t)$ is small for all t, which allows the narrowband angle modulation approximation [Ziemer 76].

$$v_m(t) = A_c(1 + \alpha(t))[\cos(\omega_c t + \phi_c) - \phi(t) \sin(\omega_c t + \phi_c)] \tag{2}$$

Convert to complex exponentials.

$$v_m(t) = \frac{A_c}{2}(1 + \alpha(t))[e^{\jmath(\omega_c t + \phi_c)} + e^{-\jmath(\omega_c t + \phi_c)} + \jmath\phi(t)(e^{\jmath(\omega_c t + \phi_c)} - e^{-\jmath(\omega_c t + \phi_c)})] \tag{3}$$

Let both the amplitude and phase modulation be complex exponentials with the same frequency, ω_m,

$$\alpha(t) = A e^{\jmath\omega_m t} \tag{4}$$

$$\phi(t) = \Phi e^{\jmath\omega_m t}, \tag{5}$$

where

$$A = A_A e^{\jmath\phi_A t} \tag{6}$$

$$\Phi = A_\Phi e^{\jmath\phi_\Phi t}. \tag{7}$$

$$v_m(t) = \frac{A_c}{2}(1+Ae^{\jmath\omega_m t})[e^{\jmath(\omega_c t+\phi_c)}+e^{-\jmath(\omega_c t+\phi_c)}+\jmath\Phi e^{\jmath\omega_m t}(e^{\jmath(\omega_c t+\phi_c)}-e^{-\jmath(\omega_c t+\phi_c)})] \tag{8}$$

Assume that both A and Φ are small and neglect cross modulation terms.

$$\begin{aligned} v_m(t) \;=\; & \frac{A_c}{2}[e^{\jmath(\omega_c t+\phi_c)} + e^{-\jmath(\omega_c t+\phi_c)} \\ & + Ae^{\jmath\omega_m t}e^{\jmath(\omega_c t+\phi_c)} + Ae^{\jmath\omega_m t}e^{-\jmath(\omega_c t+\phi_c)} \\ & + \jmath\Phi e^{\jmath\omega_m t}e^{\jmath(\omega_c t+\phi_c)} - \jmath\Phi e^{\jmath\omega_m t}e^{-\jmath(\omega_c t+\phi_c)}] \end{aligned} \tag{9}$$

$$\begin{aligned} v_m(t) \;=\; & \frac{A_c}{2}[e^{\jmath(\omega_c t+\phi_c)} + e^{-\jmath(\omega_c t+\phi_c)} \\ & + Ae^{\jmath((\omega_m+\omega_c)t+\phi_c)} + Ae^{\jmath((\omega_m-\omega_c)t-\phi_c)} \quad \leftarrow \text{AM terms} \\ & + \jmath\Phi e^{\jmath((\omega_m+\omega_c)t+\phi_c)} - \jmath\Phi e^{\jmath((\omega_m-\omega_c)t-\phi_c)}] \quad \leftarrow \text{PM terms} \end{aligned} \tag{10}$$

Rearranging

$$\begin{aligned} v_m(t) \;=\; & \frac{A_c}{2}[e^{\jmath(\omega_c t+\phi_c)} + e^{-\jmath(\omega_c t+\phi_c)} \\ & + Ae^{\jmath(\omega_m-\omega_c)t}e^{-\jmath\phi_c} - \jmath\Phi e^{\jmath(\omega_m-\omega_c)t}e^{-\jmath\phi_c} \quad \leftarrow \text{LSB terms} \\ & + Ae^{\jmath(\omega_m+\omega_c)t}e^{\jmath\phi_c} + \jmath\Phi e^{\jmath(\omega_m+\omega_c)t}e^{\jmath\phi_c}] \quad \leftarrow \text{USB terms} \end{aligned} \tag{11}$$

Assume that one performs a PAC analysis, which applies a single complex exponential signal that generates responses at the upper and lower sidebands of the ω_c signal. Assume the transfer functions are L and U, so the lower and upper sideband signals are

$$\ell(t) = Le^{\jmath(\omega_m-\omega_c)t} \tag{12}$$
$$u(t) = Ue^{\jmath(\omega_m+\omega_c)t} \tag{13}$$

where

$$L = A_L e^{\jmath\phi_L} \tag{14}$$
$$U = A_U e^{\jmath\phi_U} \tag{15}$$

Matching common frequency terms between (11), (12), and (13).

$$L = \frac{A_c}{2}(Ae^{-\jmath\phi_c} - \jmath\Phi e^{-\jmath\phi_c}) \tag{16}$$
$$U = \frac{A_c}{2}(Ae^{\jmath\phi_c} + \jmath\Phi e^{\jmath\phi_c}) \tag{17}$$

$$\frac{1}{A_c} L e^{\jmath\phi_c} = A - \jmath\Phi \tag{18}$$
$$\frac{1}{A_c} U e^{-\jmath\phi_c} = A + \jmath\Phi \tag{19}$$

Solving for the modulation coefficients gives

$$A = \frac{1}{A_c}(L e^{\jmath\phi_c} + U e^{-\jmath\phi_c}) \tag{20}$$
$$\Phi = \frac{\jmath}{A_c}(L e^{\jmath\phi_c} - U e^{-\jmath\phi_c}). \tag{21}$$

Thus, the transfer function for the amplitude modulation is given by (20), and for phase modulation it is given by (21).

1.1 Positive Frequencies

Notice that L is defined in (12) to be the transfer function from the input to the sideband at $\omega_m - \omega_c$, which is a negative frequency. This is usually a natural definition for use with SpectreRF's small signal analyses (depending on the setting of the *freqaxis* parameter), but is cumbersome when only data at positive frequencies is available. Thus, $\tilde{L}$ will be defined as the transfer function to $\omega_c - \omega_m$. Then

$$\tilde{\ell}(t) = \tilde{L} e^{\jmath(\omega_c - \omega_m)t}. \tag{22}$$

Since the signals are real, $\tilde{L}$ and L are complex conjugates of each other,

$$L = \tilde{L}^*. \tag{23}$$

and (20) and (21) are rewritten in terms of $\tilde{L}$ as

$$A = \frac{1}{A_c}(\tilde{L}^* e^{\jmath\phi_c} + U e^{-\jmath\phi_c}) \tag{24}$$
$$\Phi = \frac{\jmath}{A_c}(\tilde{L}^* e^{\jmath\phi_c} - U e^{-\jmath\phi_c}) \tag{25}$$

1.2 FM Modulation

For FM modulation, the phase modulation $\phi(t)$ becomes the integral of the FM modulation signal, $\omega(t)$.

$$v_m(t) = A_c \cos(\omega_c t + \phi(t)) \tag{26}$$

where

$$\phi(t) = \int \omega(t)dt. \tag{27}$$

Recall from (5) and (21) that

$$\phi(t) = \frac{\jmath}{A_c}(Le^{\jmath\phi_c} - Ue^{-\jmath\phi_c})e^{\jmath\omega_m t}. \tag{28}$$

Combining (27) and (28) and differentiating both sides

$$\omega(t) = \frac{\omega_m}{A_c}(Ue^{-\jmath\phi_c} - Le^{\jmath\phi_c})e^{\jmath\omega_m t}, \tag{29}$$

$$\Omega = A_\Omega e^{\jmath\phi_\Omega} = \frac{\omega_m}{A_c}(Ue^{-\jmath\phi_c} - Le^{\jmath\phi_c}) \tag{30}$$

or

$$\Omega = \jmath\omega_m\Phi. \tag{31}$$

2 Simulation

The test circuit shown in Netlists 1 and 2 was run with SpectreRF. This circuit simply consists of three linear periodically varying modulators that are driven with the same input. The input is constant valued in the large signal PSS analysis, and generates a single complex exponential analysis during the PAC analysis. The idea is to compute the transfer functions from this input to the upper and lower sidebands at the output of the modulators and then use the above derivation to convert these transfer functions into transfer functions to the AM, PM, and FM modulations and then check the results against what is expected.

Notice that *freqaxis=out*. This is necessary to match the derivation. If instead you wanted to use *freqaxis=absout*, you would have to use the complex conjugate of the L as in (24) and (25).

3 Results

The simulations were run with various values for *pacphase* on *Vin*. Table 1 shows the results for the output of the AM modulator. Table 2 shows the results for the output of the PM modulator.

Netlist 1 — AM, PM and FM conversion test circuit.

```
// AM, PM, and FM modulation test circuit

simulator lang=spectre
ahdl_include "modulators.va"

parameters MOD_FREQ=10MHz
parameters CARRIER_FREQ=1GHz

Vin (in 0) vsource pacmag=1 pacphase=0
Mod0 (unmod in) AMmodulator freq=CARRIER_FREQ mod_index=0
Mod1 (am in) AMmodulator freq=CARRIER_FREQ mod_index=1
Mod2 (pm in) PMmodulator freq=CARRIER_FREQ kp=1
Mod3 (fm in) FMmodulator freq=CARRIER_FREQ fd=MOD_FREQ

waves pss fund=CARRIER_FREQ outputtype=all tstab=2ns harms=1
xfer pac start=MOD_FREQ maxsideband=4 freqaxis=out
```

Table 1 — Result for AM modulator with $v_{\mathrm{LO}} = \cos(\omega_c t)$.

pacphase	L	U	A	Φ
0	$\frac{1}{2}$	$\frac{1}{2}$	1	0
45	$\frac{1+\jmath}{2\sqrt{2}}$	$\frac{1+\jmath}{2\sqrt{2}}$	$\frac{1+\jmath}{\sqrt{2}}$	0
90	$\frac{\jmath}{2}$	$\frac{\jmath}{2}$	$\jmath$	0
180	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	0

Table 2 — Result for PM modulator with $v_{\mathrm{LO}} = \cos(\omega_c t)$.

pacphase	L	U	A	Φ
0	$-\frac{\jmath}{2}$	$\frac{\jmath}{2}$	0	1
45	$\frac{1-\jmath}{2\sqrt{2}}$	$\frac{\jmath-1}{2\sqrt{2}}$	0	$\frac{1+\jmath}{\sqrt{2}}$
90	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\jmath$
180	$\frac{\jmath}{2}$	$-\frac{\jmath}{2}$	0	-1

Netlist 2 — Modulator models written in Verilog-A (the file *modulators.va*).

```
‘include "discipline.h"
‘include "constants.h"

module AMmodulator (out, in);
    input in;
    output out;
    electrical out, in;
    parameter real freq = 1 from (0:inf);
    parameter real mod_index = 1;

    analog begin
        V(out) <+ (1+mod_index*V(in)) * cos(2*‘M_PI*freq*$abstime);
        $bound_step( 0.05 / freq );
    end
endmodule

module PMmodulator (out, in);
    input in;
    output out;
    electrical out, in;
    parameter real freq = 1 from (0:inf);
    parameter real kp = 1 from (0:inf);

    analog begin
        V(out) <+ cos(2*‘M_PI*freq*$abstime + kp*V(in));
        $bound_step( 0.05 / freq );
    end
endmodule

module FMmodulator (out, in);
    input in;
    output out;
    electrical out, in;
    parameter real freq = 1 from (0:inf);
    parameter real fd = 1 from (0:inf);
    real phi;

    analog begin
        V(out) <+ cos(2*‘M_PI*(freq*$abstime + idtmod(fd*V(in),0,1, -0.5)));
        $bound_step( 0.05 / freq );
    end
endmodule
```

Table 3 — Result for AM modulator with $v_{\text{LO}} = \sin(\omega_c t)$.

pacphase	L	U	A	Φ
0	$\frac{\jmath}{2}$	$-\frac{\jmath}{2}$	1	0
45	$\frac{\jmath-1}{2\sqrt{2}}$	$\frac{1-\jmath}{2\sqrt{2}}$	$\frac{1+\jmath}{\sqrt{2}}$	0
90	$-\frac{1}{2}$	$\frac{1}{2}$	$\jmath$	0
180	$-\frac{\jmath}{2}$	$\frac{\jmath}{2}$	-1	0

Table 4 — Result for PM modulator with $v_{\text{LO}} = \sin(\omega_c t)$.

pacphase	L	U	A	Φ
0	$\frac{1}{2}$	$\frac{1}{2}$	0	1
45	$\frac{1+\jmath}{2\sqrt{2}}$	$\frac{1+\jmath}{2\sqrt{2}}$	0	$\frac{1+\jmath}{\sqrt{2}}$
90	$\frac{\jmath}{2}$	$\frac{\jmath}{2}$	0	$\jmath$
180	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1

If the simulations are repeated with the *cos* function in the modulators replaced by *sin*, which is equivalent to changing the LO to $v_{\text{LO}} = \sin(\omega_c t)$ or setting $\phi_c = -90$, the results shown in Tables 3 and 4 are achieved.

Finally, the results for the FM modulator are shown in Table 5. The FM modulator has a modulation coefficient of ω_m built-in, which renormalizes the results.

4 Conclusion

This paper shows that the PAC analysis can be used to determine the level of AM of PM modulation that appears on a carrier. This is done by applying

Table 5 — Result for FM modulator with $v_{\text{LO}} = \cos(\omega_c t)$.

pacphase	L	U	Ω
0	$-\frac{1}{2}$	$\frac{1}{2}$	1
45	$-\frac{1+\jmath}{2\sqrt{2}}$	$\frac{1+\jmath}{2\sqrt{2}}$	$\frac{1+\jmath}{\sqrt{2}}$
90	$-\frac{\jmath}{2}$	$\frac{\jmath}{2}$	$\jmath$
180	$\frac{1}{2}$	$-\frac{1}{2}$	-1

Netlist 3 — Ocean script that extracts and plots AM and PM transfer functions). The 3.5G in the jitter calculation is the carrier frequency.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;; Compute the AM- and PM noise according to     ;;
;; formulas (50), (51), (52), (53), (54) in      ;;
;; "KUNDERT: Introduction to RF Simulation and   ;;
;; its application", JSSC 1999                   ;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;
;;determine upper- and lower-sideband
  usb=harmonic(v("/Out" ?result "pac-pac") 1)
  lsb=harmonic(v("/Out" ?result "pac-pac") -1)
;;
;; determine the carrier:
  magCarrier=mag(harmonic(v("/Out" ?result "pss-fd.pss") 1))
  unitCarrier=harmonic(v("/Out" ?result "pss-fd.pss") 1)/magCarrier
;;
;;separate AM and PM components:
  amNoise=(lsb*unitCarrier+usb*conjugate(unitCarrier))/magCarrier
  pmNoise=complex(0. 1.)*(lsb*unitCarrier-usb*conjugate(unitCarrier))/magCarrier
;;
;; calculate the absolute amplitude variation (volts peak)
;; and jitter (seconds peak)
  amVariation=amNoise*magCarrier
  jitter=pmNoise/(2*3.1416*3.5G)
  plot(amVariation)
  plot(jitter)
;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

a small signal and using the phase of the carrier along with the transfer function to the upper and lower sidebands of the carrier to compute an AM or PM transfer function.

Finally, an Ocean script that extracts the AM and PM transfer functions from the results of a PSS/PAC analysis pair is given in Netlist 3. This script could be improved if it extracted the carrier frequency from the results rather than have it hardcoded into the script.

References

[Ziemer 76] R. Ziemer and W. Tranter. *Principles of Communications: Systems, Modulation, and Noise*. Houghton Miffin, 1976.

[Robins 96] W. Robins. *Phase Noise in Signal Sources (Theory and Application)*. IEE Telecommunications Series, 1996