

# Fast crystal-oscillator-simulation methodology

The settling behavior of a crystal oscillator typically requires days to simulate with Spice. A new methodology for fast simulation of crystal oscillators allows such simulations to proceed in less than an hour and brings benefits such as the ability to accurately predict the frequency settling behavior.

By Mark Gehring, Cypress Semiconductor

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Crystal oscillators are notoriously difficult to simulate accurately. The nature of the Newton-Raphson algorithm, which all Spice-like simulators (HSpice, PSpice, Spectre, and Eldo) use, is difficult to converge for very high Q circuits. To achieve accurate results, you must set the time step for the simulator to approximately  $T_{step} = (4/f) * \sqrt{\text{reltol}/Q}$ , where F is the oscillator frequency and reltol is the simulator's relative tolerance setting (convergence criteria) (**Reference 1**). For a 12-MHz oscillator with a Q of 40,000 and a reltol of 0.001, this equation results in a maximum time step of 52 psec. Because the settling time is on the order of several milliseconds, the simulator must produce on the order of 50 million time points, requiring sometimes days of simulation time even for a simple circuit.

## Theory

**Reference 2** describes a simulation method based on envelope analysis, which vastly simplifies the equations describing the oscillator. The essence of the method is the following:

1. The analysis starts, as in **Reference 3**, by breaking the circuit internal to the crystal equivalent circuit model as shown in **Figure 1**.
2. The authors of **references 2 and 3** point out that the current flowing in the series RLC of the crystal must be a virtually pure sinusoid, due to the high Q of the crystal. The network acts as an extremely narrow bandpass filter, removing all harmonics. This situation leads to a large simplification in the analysis. (However, the voltage across it is not so constrained, and is often rich in harmonics due to the nonlinear load of the oscillator circuit.)
3. The authors of **Reference 2** next observe that the envelope (amplitude) of the oscillations changes very slowly compared with the oscillation period, leading to further simplifications; you can reduce the entire circuit, independent of implementation, to two simple equations:

$$\begin{cases} \dot{a} = \frac{-a}{2L_q} (R_q + R_d(a)), \\ v = \dot{\phi} = -\frac{L_d(a)\omega_q}{2L_q}, \end{cases}$$

where "a" is the amplitude of the current in the RLC of the crystal,  $R_q$  is the motional resistance,  $L_q$  is the motional inductance,  $\omega_q$  is the series resonant frequency of the crystal ( $1/\sqrt{L_q C_q}$ ), and  $v$  is the instantaneous frequency of the oscillator relative to the series resonant frequency (a delta frequency).  $L_d(a)$  and  $R_d(a)$  are the nonlinear inductance and resistance, respectively, of the oscillator. They are a function of the current amplitude  $a$ .

4.  $L_d(a)$  and  $R_d(a)$  are next obtained in the following manner: The circuit in **Figure 1** is modified by replacing the series RLC of the crystal with a sinusoidal current source set to the crystal frequency  $F_q$ . The current amplitude is swept as a parameter. The top end of the sweep must be beyond the point at which the negative resistance of the oscillator

is less than  $R_q$ , because at this amplitude, the oscillator will settle. A transient (or SpectreRF PSS/EldoRF SST) analysis is run until the voltage has settled on the input node. This simulation is very fast, because there are no high Q elements in the circuit (typically a few seconds per current amplitude step, depending on circuit complexity). An FFT of the final period of the voltage waveform is computed, and the complex voltage of the fundamental (first harmonic) is divided by the current amplitude to obtain a large signal impedance. The real part is  $R_d(a)$ , and the imaginary part, divided by  $2\pi F_q$ , is  $L_d(a)$ .

**Reference 2** leaves out two important details required for simulation to proceed: the method for solving the nonlinear differential equation and the initial conditions for that equation—in other words, the initial current in the crystal at time zero.

## Implementation

The differential equation above is similar in form to an inductor in parallel with a nonlinear resistor. You can solve it conveniently with a Verilog A model, using a polynomial curve fit to the inductance and resistance as a function of the amplitude.

The initial current in the inductor  $L_q$  of **Figure 1** is obtained as the larger of the following:

A. Simulate the total integrated noise current in the series RLC with the circuit on. Because this noise is concentrated at the crystal frequency, you can consider it a single sinusoid. Multiply the results from Spice by  $\sqrt{2}$ , because “a” is the peak amplitude.

B. Do a transient simulation of the start up of the oscillator with the crystal attached. Because you are interested in only the first period of oscillation, this simulation is very fast, despite the very high Q.

Simulation of this very simple Verilog A model completes in less than one second.

## Results

This method was applied to the crystal oscillator in the CY6934 WirelessUSB crystal oscillator (**Reference 4**). The crystal was first measured with a network analyzer to obtain its exact parameters. The circuit was simulated to obtain the nonlinear resistance and inductance versus amplitude, and a sixth-order polynomial curve fit was obtained with Excel. These data were input to the Verilog A model and simulated to compare with measured and transient simulations.

**Figure 2** shows the simulated amplitude of the current in  $L_q$ . The curve on the left is the amplitude simulated with Verilog A in a fraction of a second, and the curve on the right is from a full transient simulation requiring 10.5 hours. The curves were compared at several different time points, and in all cases were identical.

**Figures 3** and **4** compare the simulation with measurement. A scope with a high-impedance ( $<0.7$  pF) probe measured amplitude versus time, and an Agilent 89441A vector signal analyzer measured frequency versus time.

Note that the new simulation method does not include harmonics of the voltage waveform, so the measurements show larger swing. For radio applications, it is important to know the settling time of the frequency. **Figure 4** shows excellent agreement ( $<1$  ppm error) between simulation and measurements.

A new method was presented for fast simulation of crystal oscillators. Comparison with full transient simulation and measurements confirm its accuracy and efficacy.

## References

1. PSpice Application Notes, pg 27, 1988.
2. Addouche et al, “Modeling of Quartz Crystal Oscillators by Using Nonlinear Dipolar Method,” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol 50, no. 5, May 2003.
3. Vittoz et al, “High Performance Crystal Oscillator Circuits: Theory and

Application," IEEE JSSC June 1988, pg 774 to 783.

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### Author's biography

Mark Gehring designed RF sections of analog cell phones at Motorola in the early 1980s, followed by circuit and system design of a wristwatch pager system at AT&E. He was a co-founder of RadioCom, which was acquired by Cypress Semiconductor in 2000, where he has been ever since, designing RF systems and circuits for the WirelessUSB chips as a senior member of the technical staff. Gehring holds 10 patents in DSP/RF. You can reach him at [gzx@cypress.com](mailto:gzx@cypress.com).

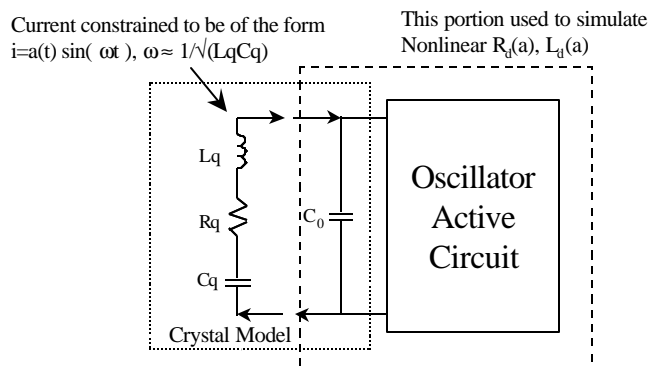


Figure 1—Generalized crystal oscillator circuit.

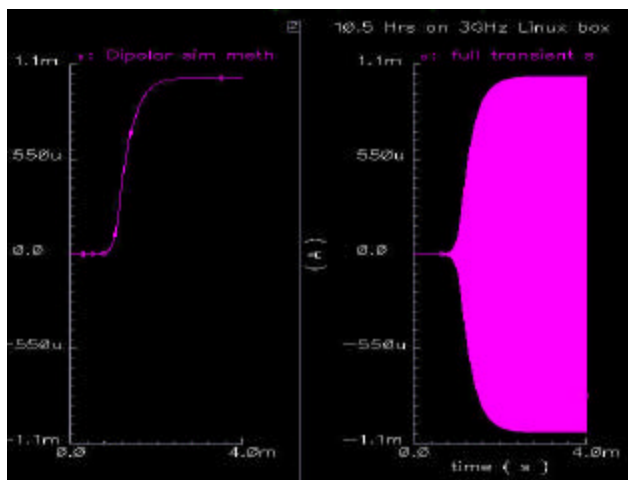
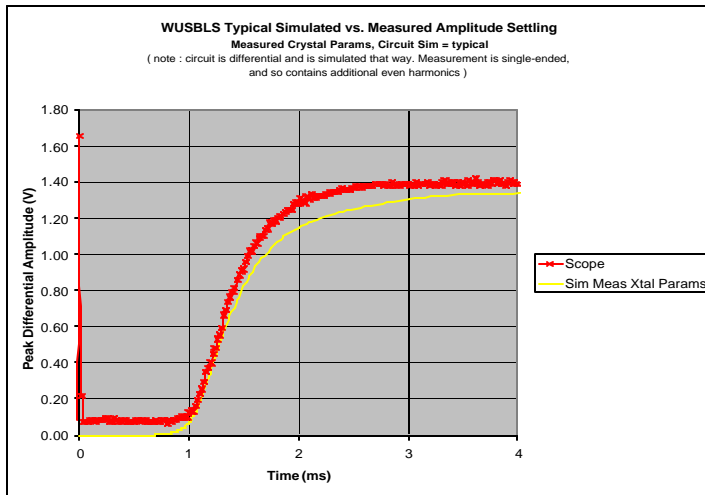
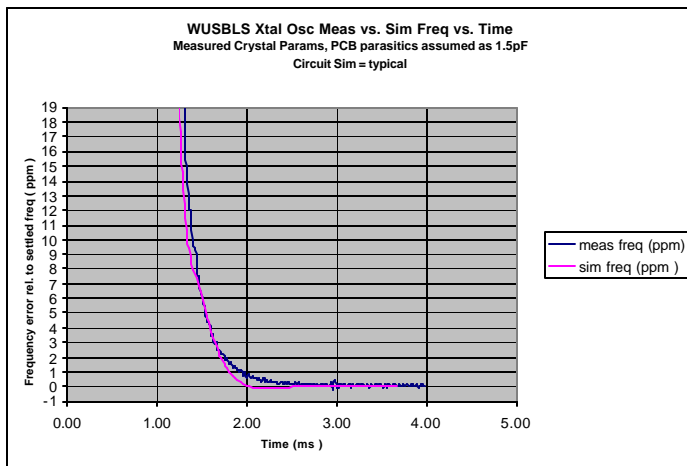


Figure 2—Inductor current versus time; full transient versus new simulation method.



**Figure 3—Measured and simulated voltage swing versus time**



**Figure 4—Measured and simulated frequency error versus time**